

ON COMPLEX-VALUED SOLUTIONS to a
2D EIKONAL EQUATION.
PART ONE: QUALITATIVE PROPERTIES

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Abstract

$w_x^2 + w_y^2 + n^2(x, y) = 0$ is a two-dimensional version of the *eikonal equation* appearing in the generalizations of geometrical optics that deal with diffraction. Here x and y denote rectangular coordinates in the Euclidean plane, and n is real-valued. A framework is proposed, which consists of Bäcklund transformations and second-order PDEs governing $\operatorname{Re}(w)$ and $\operatorname{Im}(w)$. Sample solutions are constructed in the case where n is constant. The critical points of $\operatorname{Re}(w)$ are the main motif. Theorems, focusing on the geometry of such critical points, are given.