ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Institut d'Analyse et Calcul Scientifique (IACS)
Section Mathématiques

SEMINAIRE D'ANALYSE

- ***VENDREDI 28 novembre 2008 à 16h00 à la salle MA A112***

Professeur ***Paolo MARCELLINI*** *(University of Firenze - Italy)* donnera une conférence sur le thème:

"WEAK LOWER SEMICONTINUITY FOR NON COERCIVE POLYCONVEX INTEGRALS"

Abstract: Dealing with vector-valued maps, still it is not completely known a set of minimal assumptions for lower semicontinuity of integrals of the calculus of variations of the form

$$F(u) = \int_\Omega g(x, u, Du)\, dx \ ,$$

where $u : \Omega \to \mathbb{R}^m$ is a *vector-valued map* defined in an open set $\Omega \subset \mathbb{R}^n$ and Du is the $m \times n$ *Jacobian matrix* of its partial derivatives

$$u \equiv \left(u^1, u^2, \ldots, u^m\right), \qquad Du = \left(\frac{\partial u^\alpha}{\partial x_i}\right)_{i=1,2,\ldots,n}^{\alpha=1,2,\ldots,m} .$$

On the contrary, in the so-called *scalar case* (corresponding to $m = 1$) Serrin in 1961, in a pioneering paper, pointed out the convexity of $g = g(x, s, \xi)$ with respect to the gradient variable ξ as a main (necessary and) sufficient condition for the lower semicontinuity of the integral $F(u)$. We will recall Serrin's results on the lower semicontinuity of F with respect to the $L^1_{\text{loc}}(\Omega)$-convergence. In the vector-valued case $m > 1$ either the *quasiconvexity* or the *polyconvexity* of g with respect to the gradient variable ξ play a role. These convexity conditions are due to Morrey. In particular the function $g(x, s, \xi)$ is said *polyconvex* with respect to the gradient variable ξ if it can be represented under the form

$$g(x, s, \xi) = f(x, s, M(\xi)) \ ,$$

where f is a *convex* function with respect to its last variable and, for every $m \times n$ matrix $\xi \in \mathbb{M}^{m\times n}$, $M(\xi)$ denotes the vector

$$M(\xi) = \left(\xi, \operatorname{adj}_2\xi, \ldots, \operatorname{adj}_i\xi, \ldots, \operatorname{adj}_{\min\{m,n\}}\xi\right) .$$

The lower semicontinuity for polyconvex integrals have been investigated by several authors in the past years. We present here some new lower semicontinuity results for polyconvex functionals of integral form, related to maps $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ in $W^{1,n}(\Omega; \mathbb{R}^m)$ with $n \geq m \geq 2$, with respect to the weak $W^{1,p}$-convergence for $p > m - 1$, without assuming any coercivity condition.

Lausanne, novembre 2008
BD

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