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Su una convergenza di funzioni convesse. (Italian. English summary)

. Boll. Un. Mat. Ital. (4) **8** (1973), 137–158.

Let V be a real reflexive Banach space with dual V^*. The author denotes by $\tilde{\mathcal{C}}(V)$ the totality of continuous convex functions $f\colon V \to \mathbf{R}$ such that, for every $v^* \in V^*$, the function $v \mapsto f(x) + \langle v^*, v\rangle$ attains its infimum. Equivalently, the conjugate convex function f^* belongs to $\tilde{\mathcal{C}}(V^*)$. Two elements α_1 and α_2 of $\tilde{\mathcal{C}}(V)$ being fixed, the set of convex functions f satisfying the condition $\alpha_1 \leq f \leq \alpha_2$ is denoted by $\tilde{C}[\alpha_1, \alpha_2]$. The pointwise ordering of numerical functions induces on $\tilde{\mathcal{C}}[\alpha_1, \alpha_2]$ an ordering for which this set is a complete lattice. Making use of this lattice structure, the author introduces, for a sequence (f_n) of elements of $\tilde{\mathcal{C}}[\alpha_1, \alpha_2]$, the two concepts of C-convergence and G-convergence toward some $f \in \tilde{\mathcal{C}}[\alpha_1, \alpha_2]$, which turn out to be equivalent to the following: $f = G\text{-Lim}\, f_n$ if and only if f^* is the pointwise limit of $({f_n}^*)$; $f = C\text{-Lim}\, f_n$ if and only if f and f^* are, respectively, the pointwise limits of (f_n) and $({f_n}^*)$; in addition, for $f_n \in \tilde{\mathcal{C}}[\alpha_1, \alpha_2]$, the C-convergence is equivalent to U. Mosco's convergence [Advances in Math. **3**, 510–585 (1969); MR0298508 (45 #7560)]. The C- and G-convergence are equivalent when $\dim V < +\infty$. If $f = G\text{-lim}\, f_n$ is strictly convex and if u_n is a minimizing point of f_n, the sequence (u_n) is weakly convergent to the minimizing point of f. If V is separable, every sequence in $\tilde{\mathcal{C}}[\alpha_1, \alpha_2]$ possesses a G-convergent subsequence.

A convex function on V is said to be polyhedral if it is the pointwise supremum of a finite family of continuous affine functions; by definition a quasi-polyhedral convex function is the limit, uniformly on every bounded subset of V, of some sequence of polyhedral convex functions. Let (f_n) be a sequence in $\tilde{\mathcal{C}}[\alpha_1, \alpha_2]$ and $({f_n}')$ be a sequence of convex functions such that $f_n + {f_n}' \in \tilde{\mathcal{C}}[{\alpha_1}', {\alpha_2}']$; if $f = G\text{-Lim}\, f_n$ and if $({f_n}')$ converges, uniformly on every bounded subset of V, to a quasi-polyhedral function f', then $G\text{-Lim}(f_n + {f_n}') = f + f'$.

Some functional examples involving differential operators are given.

Reviewed by *J. J. Moreau*