

CIME Course and Partial Differential Equations and Calculus of Variations

Elvira Mascolo

David Hilbert used to say

every real progress walks **hand in hand** with the discovery of more and more rigorous tools and simpler methods which meanwhile make easier the understanding of previous theories.

Nevertheless Augustus De Morgan used to say:

The mental attitude which stimulate the mathematical invention is not only a sharp reasoning but rather a deep imagination.

The “progress” Hilbert was talking about is based – in mathematics more than in other scientific fields – upon teaching and collaboration and the “imagination” De Morgan was referring to, must be stimulated through a progressive and gradual learning.

Both history and everybody personal experience show that mathematical learning and its improvement is not just a matter of studying books and original articles, but rather that of a continuous and effective relationships with our own teacher(s), rising new questions and discussing together their possible answers.

In the Fifties a group of outstanding Italian mathematicians, all member of the Scientific Committee of UMI (the Union of Italian Mathematicians) under the presidency of Enrico Bompiani, decided that it was the moment to rise the mathematical research in Italy to the level it was before the Second Worldly War and that it should be done through the organisation of high level courses. They realized the importance of providing the young researchers with the possibility of learning the new theories, subjects and themes which were appearing in those years and of mastering the new techniques and tools.

It was right in those years that the CIME was founded and the first course was held in Varenna (a charming small city on the Como lake) in 1954. The subject was on Functional Analysis, which can be considered at that time a new subject. More precisely:

Funzionali Analitici Ed Anelli Normati

Varenna (Como), June, 9–18.

Lectures L.Amerio (Politecnico Milano), L.Fantapié (Univ. Roma), E.R.Lorch (Columbia Univ.)

Seminars: M.Cugiani (Univ. Milano), F.Pellegrino (INAM, Roma), G.B.Rizza (Univ. Genova).

The second was also held in Varenna in the August of the same year. That was on

Quadratura Delle Superficie e Questioni Connesse

Varenna (Como), August, 16–25.

Lectures: R. Caccioppoli (Univ. Napoli), L. Cesari, (Univ.Bologna, Purdue Univ.), Chr. Y. Pauc (Univ. Rennes).

Seminars: A. Finzi (Technion, Haifa), A.Zygmund (Univ. Chicago).

The exceptional personalities were the teachers chosen in that occasion: Renato Caccioppoli, Lamberto Cesari and Antoni Zygmund. How can we recall without the suspect of being limited the contributions given for example by Caccioppoli to the development of the modern Functional Analysis and the Geometric Theory of Measure.

In the subsequent fifty-one years the CIME organized 165 courses which cover basically every aspect of mathematics, both pure and applied, thus playing a crucial role in promoting and developing the mathematical research and not only in Italy.

In fact the CIME activities have favoured and promoted personal contacts among distinguished scientists and young researchers.

The mathematics in last years has known a nearly explosive development and the organization of courses is an exceptional instrument of formation for the young investigators and a real support for the most mature ones.

The *full immersion* permitted by a common location was the right preamble to develop new subjects, to suggest new methods, to learn how to apply old methods to new problems, to start joint papers.

One main reason for the success of CIME courses was in particular the fact that they have been all published and the CIME Sessions are an essential mean of diffusion of the mathematical culture.

The texts of lectures and seminars of each Session were all published:

- The volumes of Sessions 1-39 are actually out of print,
- The volumes of Sessions 39-70 are on the Catalogue of Edizioni Cremonese, Firenze, Italy
- The volumes of Sessions 71-83 are on the Catalogue of Liguori Editore, Napoli, Italy
- Since 1981 all courses notes are being published by Springer Verlag in a Subseries (Fondazione C.I.M.E.) of the Lectures Notes in Mathematics

My aim today is to guide you in a ideal journey through the development of the Calculus of Variation ad Nonlinear Differential Equation via the CIME courses held on these topics in the last fifty years of his history.

For the majority of younger people some arguments and techniques are to be considered as standard; however in these fifty years the developments have been so fast than the so-called “variational questions” raised by Hilbert at the beginning of the past century, have blowed up during the entire century (particularly the second half), in so many and different directions that Hilbert himself could have never imagined.

Quoting James Serrin we could say that

the relevant field of investigation is nowadays so spread and wide that only a few years ago would have thought of as unbelievable.....more new ideas and results appeared from the end of the Second Worldly War until the present day than from the time of Talete until the year 1945.

- Strictly related with the variational question was the CIME Session of 1958 in which Alessandro Faedo was the Director of a session devoted to the minimum principles and their applications.

Principio di Minimo e le sue Applicazioni in Analisi Funzionale

Pisa, September 1–10.

Director: S.Faedo (Univ. Pisa)

Lectures: L. Bers (Courant Institute), Ch. B.Morrey, Jr. (Univ. Of California, Berkeley), L.Nirenberg (Courant Institute).

Seminars: S.Agmon (Hebrew Univ. Jerusalem), G.Fichera (Univ. Roma), G.Stampacchia (Univ. Genova).

Notice the presence as lectures of Charles Jr. Morrey. and Louis Nirenberg, two of the most important among the personalities in the Calculus of Variations and Partial Differential Equations of past century. The notes are published on Annali Scuola Normale di Pisa.

- Under the direction of Enrico Bompiani the session of 1961 represents a first approach to the geometric methods in the Calculus of Variations.

Geometria del Calcolo delle Variazioni

Saltino (Firenze), August, 21–30.

Director: E.Bompiani (Univ. Roma).

Lectures: H.Busemann (Univ. of Southern Calif., Los Angeles), E.T.Davies (Univ. Southampton), D.Laugwitz (Technische Hochschule, Darmstadt).

- In 1964, Guido Stampacchia was the Scientific Director of a very important session, in which we note the relevance not only the lecturers but also of the young researchers which gave a seminar.

Equazioni Differenziali non Lineari

Varenna (Como), August 30 – September 8.

Director: G. Stampacchia (Univ. Pisa).

Lectures: P. Lax (New York Univ.), J. Leray (Collège De France), J. Moser (New York Univ.).

Seminars: R. Courant (New York Univ.), E. Degiorgi (Scuola Normale Superiore, Pisa), J. Friberg (Univ. Lund), J. Necas (CSAV, Praha), I. Segal (M.I.T.), G. Stampacchia (Univ. Pisa) O. VeJVoda (CSAV, Praha).

Unfortunately there is not the text of the seminar given by Ennio de Giorgi.

- In 1966 Roberto Conti, one of the most important Italian mathematician and in some sense the “father” of CIME (he was Scientific Secretary since 1954 to 1974 and the Director of CIME Foundation since 1975 to 1998) was the director of this session dedicated to the applications of the methods of Calculus of Variation to Control Theory that at that times is called “modern” Calculus of Variations.

Calculus of Variations, Classical and Modern

Bressanone (Bolzano), June, 10–18.

Director: R. Conti (Univ. Firenze).

Lectures: A. Blaquiere (Univ. Paris-Orsay), L. Cesari (Univ. Michigan), E. Rothe (Univ. Michigan), E.O. Roxin (Univ. Buenos Aires)

Seminars: C. Castaing (Univ. Caen), H. Halkin (univ. of California, La Jolla), C. Olech (Univ. Krakow).

- In 1972 Enrico Bombieri was the director of this session dedicated to the Geometric measures. The lecturers are one of the most important in the field: Enrico Giusti, Frederik Almgren and Mario Miranda.

Geometric Measure Theory and Minimal Surfaces

Varenna (Como), August 25 – September 2.

Director: E. Bombieri (Univ. Pisa).

Lectures: W.K. Allard (Princeton Univ.), F.J. Almgren (Princeton Univ.), E. Bombieri, E. Giusti (Univ. Pisa), M. Miranda (Univ. Ferrara).

Seminars: J. Guckenheimer (Princeton Univ.), D. Kinderlehrer (Univ. of Minnesota), L. Piccinini (SNS, Pisa).

- Guido Stampacchia and Gianfranco Capriz were, in 1972, the scientific directors of this important session on the Variational Methods in Mathematical Physics, which at that time were called “new”.

New Variational Techniques in Mathematical Physics

Bressanone (Bolzano), June, 17–26.

Directors: G. Capriz (Univ. Pisa), G. Stampacchia (SNS, Pisa).

Lectures: G. Duvaut (Univ. Paris XIII), J.J. Moreau (Univ. Languedoc, Montpellier), B. Nayroles (Univ. Poitiers).

Seminars: C. Baiocchi (Univ. Pavia), Ch. Castaing (Univ. Languedoc, Montpellier), D. Kinderlehrer (Univ. of Minnesota), H. Lanchon (Univ. of Essex), J.M. Lasry (univ. Paris-dauphine), W. Noll (Carnegie Mellon Univ.), W. Velte (Univ. Wurzburg).

- In 1984 Enrico Giusti was the director of that session on the Harmonic Mapping.

Harmonic Mappings and Minimal Immersions

Montecatini Terme (Pistoia), June 24 – July 3.

Director: E. Giusti (Univ. Firenze).

Lectures: S. Hildebrandt (Univ. Bonn), J. Jost (Univ. Bonn), L. Simon (Australian Nat. Univ., Canberra).

Seminars: J.H. Sampson (Johns Hopkins Univ.), M. Seppala (Univ. Helsinki)

- In 1987 Mariano Giaquinta was the director of this session in which different aspects of the Calculus of Variations were presented. In this session for the first time Luis Caffarelli was a lecture. It is the begin of his valuable collaboration with CIME.

Topics in Calculus of Variations

Montecatini Terme (Pistoia), July, 20–28.

Director: M. Giaquinta (Univ. Firenze).

Lectures: L. Caffarelli (IAS, Princeton), A. J. Moser (ETH, Zurich), L. Nirenberg (Courant Inst.), R. Schoen (Univ. of California, San Diego), A. Tromba (Max-Planck Inst., Bonn).

- In 1989 Arrigo Cellina was the director of this session devoted to non convex problems and the methods for studying different subjects. In particular the non convex functionals of Calculus of Variations were considered.

Methods of Nonconvex Analysis

Varenna (Como), June, 15–23.

Director: A. Cellina (SISSA, Trieste)

Lectures: I. Ekeland (Univ. Paris, Dauphine), P. Marcellini (Univ. Firenze), A. Marino (Univ. Pisa), C. Olech (PAN, Warszawa), G. Pianigiani (Univ. Siena), T.R. Rockafellar (Univ. of Washington, Seattle), M. Valadier (USTL, Montpellier).

- I. Capuzzo Dolcetta and P. L. Lions were in 1995 the directors on this session devoted to the viscosity solution and their applications in several fields of Partial Differential Equations. One of the lecturers was L. C. Evans, which in the subsequent years has participated several times in the CIME activities.

Viscosity Solutions and Applications

Montecatini Terme (Pistoia) June, 12-20,

Director: I. Capuzzo Dolcetta (Univ. Roma, La Sapienza) P. L. Lions
(Univ. Paris Dauphine)

Lectures

M. Bardi (Univ. Padova), M. G. Crandall (Univ. California, Santa
Barbara), L. C. Evans (Univ. California, Berkeley), M. H. Sonner
(Carnegie Mellon Univ.), P. E. Souganidis (Univ. Wisconsin).

- In 1996 S. Hildebrandt and M. Struwe were directors of the session, which gave a review of the different contributions to variational methods for Ginzburg-Landau equations, for microstructure and phase transitions and for the Plateau Problem.

Calculus of Variations and Geometric Evolution Problems

Cetraro (Cosenza), June 15-22.

Directors: S. Hildebrandt (Univ. Bonn), M. Struwe (ETH, Zurich).

Lectures: F. Bethuel (ENS, Cachan), R. Hamilton (Univ. of California,
San Diego), S. Muller (ETH, Zurich), K. Steffen (Univ. Dusseldorf).

- Luis Caffarelli with Sandro Salsa in 2001 were the directors of this session which, with different point of views and perspectives, represents a basic guide on Optimal Transportation.

Optimal Transportation and Applications

Martina Franca (Taranto), Sept. 2-8.

Directors: L. Caffarelli (Univ. of Texas, Austin), S. Salsa (Politecnico
Milano).

Lectures. L. Caffarelli (Univ. of Texas, Austin), G. Buttazzo (Univ.
Pisa), L. C. Evans (Univ. of California, Berkeley), Y. Brenier
(LAN-UPMC, Paris VI), C. Villani (Ecole Normale Sup., Lyon).

- Bernard Dacorogna and Paolo Marcellini are the directors of this session, with more than 150 participants, one of the course more numerous of the last years.

This means that the Calculus of Variations is a still interesting and alive subject.

**Calculus Of Variations And Non Linear Partial Differential
Equations**

June 27 - July 2, 2005 - Cetraro (Cosenza)

Directors: Bernard Dacorogna (EPLF, Lousanne) Paolo Marcellini,
(Univ. Firenze)

Lectures L. Ambrosio (SNS Pisa), Luis A. Caffarelli (Univ. of Texas,
Austin), M. Crandall (Univ. California, Santa Barbara), L. C. Evans
(Univ. California, Berkeley, Usa), G. Dal Maso (SISSA, Trieste), N.
Fusco (Univ. Napoli)

Let me conclude with the opinion of Cantor

The essence of mathematics is freedom and *independence*...

freedom expressed as driving curiosity of a bright child....

freedom to pursue innocent fascination until it finally touched the
world we all live in.