Minicourse on
"Algebraic cycles in families and the standard conjectures"

The course, aimed to doctoral students or researchers interested in Algebraic Geometry, will be held by

Mattia Cavicchi (Université Paris-Saclay)

at Dipartimento di Matematica e Informatica Ulisse Dini in Florence,
viale Morgagni 67/A, in three afternoons,

Monday February 19 , sala conferenze Tricerri, 14.30 - 16.30
Wednesday February 21, sala conferenze Tricerri, 14.30 - 16.30
Friday February 23, room 207, 14.30 - 16.30

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Abstract

Given a complex, smooth projective variety, one can study it through invariants of different nature. A natural bridge from invariants of algebraic-geometric nature (groups of algebraic cycles) to invariants of topological nature (singular cohomology) is provided by the cycle class map. One of the longstanding open problems of algebraic geometry consists precisely in understanding the image of the cycle class map, and Grothendieck's standard conjectures point out some natural classes that should lie in it. The goal of this three-lecture series is to give an introduction to this circle of ideas and to explain the recent proof of new cases of the standard conjectures, through an approach based on studying how the desired classes behave when algebraic varieties move in families, with possibly singular fibers (joint work of the lecturer with G. Ancona, R. Laterveer and G. Saccà).

- Lecture I - "Algebraic cycles and the standard conjectures"
In the first lecture, we will give a quick review of the relation between algebraic cycles and singular cohomology on a complex, smooth projective variety, and explain the statements and the relevance of Grothendieck's standard conjectures on algebraic cycles. We will focus in particular on the standard conjecture of Lefschetz type L(X) for a smooth projective X, which implies all the other standard conjectures on X when one works over the complex numbers.

- Lecture II - "Projective families of algebraic varieties and the decomposition theorem"
In the second lecture, we will move to the relative setting and explain the additional structures existing on the cohomology of a smooth projective variety, when it admits a fibration in projective varieties over a base. We will discuss relative cycles and the BBDG decomposition theorem for a projective morphism f, and use these to formulate a relative version, denoted L(f), of the Lefschetz standard conjecture.

- Lecture III - "Standard conjectures under fibration and an application to hyperkähler varieties"
In the final lecture, we will consider the problem of proving the Lefschetz standard conjecture L(X) for a smooth projective variety X, when there exists a projective morphism f:X->B such that L(B) and L(f) are known. We will discuss the obstacles arising when trying this approach and explain how they can be overcome in the realm of hyperkähler varieties equipped with a Lagrangian fibration, in order to obtain a proof of the standard conjectures for new classes of such varieties.