

Mathematical Physics: A mathematical model of frying processes

A. Mancini

Univ. di Firenze, Dipartimento di Matematica “U.Dini”
Firenze - Italy
alberto.mancini@math.unifi.it

M. Primicerio

Univ. di Firenze, Dipartimento di Matematica “U.Dini”
Firenze - Italy
mario.primicerio@math.unifi.it

Yiqing Yang

Hangzhou Dianzi Univ., Department of Mathematics
Hangzhou 310018, China

Abstract

We present a mathematical model for the process of frying a rather thick sample of an indeformable porous material saturated with water (e.g. a potato slice).

The model is based on thermodynamical arguments and results in a initial-boundary value problem for a system of equations satisfied by the temperature and vapour content, with a free boundary separating the region saturated with water and the vapour region.

We provide some results of numerical simulations.

In questo lavoro viene presentato un modello per il processo di frittura per immersione. Si considera il processo applicato ad un campione il cui spessore sia sufficientemente grande da rendere trascurabili le deformazioni dovute alla cottura come accade, per esempio, nelle comuni patatine fritte.

Lo sviluppo del modello è basato su considerazioni termodinamiche ed ha la forma di un sistema di equazioni differenziali alle derivate parziali con frontiera libera, quest'ultima rappresentata dal fronte di desaturazione dovuta alla vaporizzazione dell'acqua contenuta nel campione (modellato come un mezzo poroso non deformabile). Le incognite del sistema rappresentano la temperatura all'interno del campione ed il vapore contenuto.

Keywords and Phrases: mathematical model, frying, moving boundary

1 Introduction

In the last decade, several papers have been devoted to the mathematical model of frying, starting from [2] [3] that abandoned the purely phenomenological approach of previous attempts and analyzed and discussed the different phenomena of mass and energy transfer that are involved in the process.

Referring to one-dimensional geometry with planar symmetry, a slab $x \in (-L, L)$ of porous material saturated with water and having a given temperature is considered to be put in contact, on the faces $x = \pm L$, with an oil bath kept at temperature T_∞ above the boiling point of water. This is assumed to happen starting from time $t = 0$ and frying is described as a coupling between heat transfer (conduction and convection) and vapour migration in an undeformable porous medium (see [1] and [8]).

As a matter of fact, this idealized situation exhibits most of the basic ingredients that seem to be relevant e.g. in the process of frying relatively thick samples of potatoes. Moreover it is suitable for rather easy numerical simulation and hence to possible experimental validation. Of course, once this preliminary check is obtained, modification induced in the organic material by the thermal history (see [8] [9] and e.g. [7] for an introduction to the influence of the process on the result) as well as more realistic geometry will be taken into account.

But, even in the idealized situation described above, the problem is far from being trivial. In a recent paper [5] a complete analysis of the phenomenon, based on a correct application of the basic balance laws, has been performed. There the mathematical model is formulated in terms of a non-standard free boundary problem for a system of parabolic equations. Indeed, the region $x \in (-L, L)$, $t > 0$ (of course, using symmetry the analysis is confined to $(0, L) \times \mathbb{R}^+$) is the union of four sub-region separated by free boundary: the water-saturated part, the region of coexistence of vapour and water, the pure vapour region and the crust.

Here, we present a simplified model in which the region of coexistence is assumed to have a negligible thickness, while incorporating in the model the correct Rankine-Hugoniot type conditions. Moreover, in the model that will be presented and discussed, crust formation is neglected although a simple modification of the boundary condition on $x = L$ is suggested in order to take this effect into account.

Thus, only one free boundary and two regions are considered: region 1, where the porous medium is completely saturated by water, and region 2 in which the pores are filled by water vapour in thermodynamical equilibrium.

A crucial point is to assume that the porous medium is non-deformable and thus frying processes of thin layers (e.g. tortilla chips, see [9]) will need a substantially different model.

After evaluating the time scales, it will be possible to reach a partial disentangling of the problem for the unknown temperature from the problem for the unknown pressure, suggesting how to investigate the well-posedness of the problem.

Finally, numerical simulations are shown for some specific cases.

2 The governing equations and the boundary conditions

According to the discussion of the previous section, we write the balance equations in the two regions.

In region 1 (water-saturated porous medium), we have that pressure is constant, since $\frac{\partial p}{\partial x} = 0$ at $x = 0$ and the medium is assumed non-deformable (and water compressibility is neglected). Thus, the only equation we have to consider is the energy balance

$$(\rho c)_1 \frac{\partial T}{\partial t} - k_1 \frac{\partial^2 T}{\partial x^2} = 0, \quad (2.1)$$

where $(\rho c)_1$ and k_1 are respectively heat capacity and conductivity in region 1, that are essentially given by

$$(\rho c)_1 = (1 - \varepsilon)\rho_s c_s + \varepsilon\rho_w c_w, \quad (2.2)$$

$$k_1 = (1 - \varepsilon)k_s + \varepsilon k_w. \quad (2.3)$$

In (2.2) and (2.3), we denoted by ε the porosity and ρ , c and k represent density, specific heat and conductivity respectively, while suffixes s and w refer to solid matrix and water, respectively.

In region 2 (solid + vapour) we have to consider mass balance in addition to energy conservation. Indeed, vapour migration is induced by pressure gradient, so that Darcy's law implies

$$\varepsilon \frac{\partial \rho_v}{\partial t} - \frac{K}{\mu} \frac{\partial}{\partial x} \left(\rho_v \frac{\partial p}{\partial x} \right) = 0, \quad (2.4)$$

where suffix v refers to vapour and K , μ denote intrinsic permeability of the porous medium and dynamic viscosity of vapour.

Passing to energy balance, we should consider that, in principle, heat capacity and conductivity depend on ρ_v , but it is reasonable to assume that

$$(\rho c)_2 = (1 - \varepsilon)\rho_s c_s + \varepsilon\rho_v c_v \approx (1 - \varepsilon)\rho_s c_s, \quad (2.5)$$

$$k_2 = (1 - \varepsilon)k_s + \varepsilon k_v(\rho_v) \approx (1 - \varepsilon)k_s. \quad (2.6)$$

The most important difference with energy balance in region 1 is that now a relevant role is played by convection. Indeed, we have

$$(\rho c)_2 \frac{\partial T}{\partial t} - k_2 \frac{\partial^2 T}{\partial x^2} - c_v \frac{K}{\mu} \frac{\partial}{\partial x} \left(\rho_v \frac{\partial p}{\partial x} (T - T_0) \right) = 0, \quad (2.7)$$

where T_0 is the boiling point temperature at atmospheric pressure.

Let us pass to discuss the conditions on the external boundary and at the initial time. For the latter, we have e.g.

$$T(x, 0) = \bar{T} < T_0, \quad 0 < x < L, \quad (2.8)$$

where we may allow, in general, \bar{T} to depend on x . Moreover, for symmetry reasons,

$$\frac{\partial T}{\partial x}(0, t) = 0, \quad t > 0. \quad (2.9)$$

On the boundary $x = L$, we write

$$-k_1 \frac{\partial T}{\partial x}(L, t) = \gamma_1(T(L, t) - T_\infty), \quad 0 < t < t^*, \quad (2.10)$$

and

$$-k_2 \frac{\partial T}{\partial x}(L, t) = \gamma_2(T(L, t) - T_\infty), \quad t > t^* \quad (2.11)$$

where T_∞ is the (prescribed) temperature of the oil bath, γ_i are thermal exchange coefficients (in principle, γ_2 is dependent on the thermal history of the surface and on the discharge of vapour) and

$$t^* = \sup_t \{T(L, t) < T_0\}$$

Moreover for $t > t^*$

$$p(L, t) = p_0, \quad t > t^*, \quad (2.12)$$

where p_0 is the atmospheric pressure.

Finally, we have to write the conditions on the free boundary $x = s(t)$, $t > t^*$, separating the two regions. To simplify notation we write $T^\pm \equiv T(s(t) \pm 0, t)$ and similarly for the other quantities. Thus, we have

$$T^- = T_0, \quad (2.13)$$

Pressure is continuous and given by the Clapeyron's law

$$p^+ = p^- = p_0 \exp \left\{ \frac{\lambda}{R} \left(\frac{1}{T_0} - \frac{1}{T^+} \right) \right\}, \quad (2.14)$$

where λ is the latent heat of vapourization and R is the gas constant. Note that we had to write on the r.h.s. the temperature of vapour, since temperature will be discontinuous across the free boundary, as a consequence of the fact that in our scheme the thickness of the region occupied by saturated vapour is neglected.

Imposing global mass and energy balance and differentiating as in the procedure currently used to obtain Rankine-Hugoniot jump conditions, we obtain

$$\varepsilon(\rho_w - \rho_v^+)s'(t) = \frac{K}{\mu}\rho_v^+p_x^+, \quad t > t^* \quad (2.15)$$

that will be approximated by

$$\varepsilon\rho_w s'(t) = \frac{K}{\mu}\rho_v^+p_x^+, \quad t > t^* \quad (2.16)$$

$$-\varepsilon\lambda\rho_w s'(t) + k_1 T_x^- - k_2 T_x^+ - (\rho c)_2 (T^+ - T_0) s'(t) - c_v (T^+ - T_0) \frac{K}{\mu} \rho_v^+ p_x^+ = 0, \quad t > t^*, \quad (2.17)$$

We will finally assume that, in region 2, ρ_v , p and T are related by a known state equation. For simplicity, we take

$$p = RT\rho_v. \quad (2.18)$$

Of course, the "initial" condition for $s(t)$ is

$$s(t^*) = L. \quad (2.19)$$

3 Rescaling

We set

$$x = \xi L, \quad (3.1)$$

$$T = u T_0, \quad (3.2)$$

$$p = v p_0, \quad (3.3)$$

and we rescale time by a constant σ that will be chosen later

$$t = \sigma \tau. \quad (3.4)$$

Then, we define

$$\rho_0 = p_0 / (RT_0), \quad (3.5)$$

and

$$\tau^* = t^*/\sigma, \quad (3.6)$$

so that $\tau^* = \sup_{\tau} \{u(1, \tau) < 1\}$. We also define the rescaled free boundary between the water-saturated region and the vapour region

$$z(\tau) = \begin{cases} 1, & \tau \leq \tau^*, \\ s(\sigma\tau)/L, & \tau > \tau^* \end{cases} \quad (3.7)$$

Then, after simple calculation, we get

$$\frac{\partial u}{\partial \tau} = \frac{\sigma}{t_1} \frac{\partial^2 u}{\partial \xi^2}, \quad 0 < \xi < z(\tau), \quad \tau > 0, \quad (3.8)$$

$$\frac{\partial u}{\partial \tau} = \frac{\sigma}{t_2} \frac{\partial^2 u}{\partial \xi^2} + \varepsilon \frac{\rho_0 c_v}{(\rho c)_2} \frac{\sigma}{t_3} \frac{\partial}{\partial \xi} \left[v \frac{\partial v}{\partial \xi} \left(1 - \frac{1}{u} \right) \right], \quad z(\tau) < \xi < 1, \quad \tau > \tau^*, \quad (3.9)$$

$$\frac{\partial v}{\partial \tau} - \frac{v}{u} \frac{\partial u}{\partial \tau} = \frac{\sigma}{t_3} u \frac{\partial}{\partial \xi} \left[\frac{v}{u} \frac{\partial v}{\partial \xi} \right], \quad z(\tau) < \xi < 1, \quad \tau > \tau^*, \quad (3.10)$$

where

$$t_1 = \frac{L^2(\rho c)_1}{k_1}, \quad (3.11)$$

$$t_2 = \frac{L^2(\rho c)_2}{k_2}, \quad (3.12)$$

$$t_3 = \frac{L^2 \varepsilon \mu}{p_0 K}. \quad (3.13)$$

Initial and fixed boundary conditions read

$$u(\xi, 0) = \bar{u}(\xi) = \bar{T}(\xi)/T_0, \quad 0 < \xi < 1, \quad (3.14)$$

$$\frac{\partial u}{\partial \xi}(0, \tau) = 0, \quad \tau > 0, \quad (3.15)$$

$$-\frac{\partial u}{\partial \xi}(1, \tau) = \Gamma_1 [u(1, \tau) - u_\infty], \quad 0 < \tau < \tau^*, \quad (3.16)$$

$$-\frac{\partial u}{\partial \xi}(1, \tau) = \Gamma_2 [u(1, \tau) - u_\infty], \quad \tau > \tau^* \quad (3.17)$$

$$v(1, \tau) = 1, \quad \tau > \tau^*. \quad (3.18)$$

where $\Gamma_1 = \gamma_1 L/k_1$, $\Gamma_2 = \gamma_2 L/k_2$, $u_\infty = T_\infty/T_0$.

Next, we consider the interphase conditions. From (2.16), we have

$$\frac{dz}{d\tau} = \frac{\sigma}{t_3} \frac{\rho_0}{\rho_w} \frac{v^+}{u^+} \left(\frac{\partial v}{\partial \xi} \right)^+, \quad \tau > \tau^*, \quad (3.19)$$

From (2.17), we obtain

$$\frac{dz}{d\tau} = \frac{(\rho c)_1 T_0}{\varepsilon \lambda \rho_w} \frac{\sigma}{t_1} u_\xi^- - \frac{(\rho c)_2 T_0}{\varepsilon \lambda \rho_w} \frac{\sigma}{t_2} u_\xi^+ - \frac{(\rho c)_2 T_0}{\varepsilon \lambda \rho_w} (u^+ - 1) \frac{dz}{d\tau} - \frac{\rho_0 c_v T_0}{\lambda \rho_w} \frac{\sigma}{t_3} v^+ v_\xi^+ \frac{u^+ - 1}{u^+}, \quad (3.20)$$

$$\tau > \tau^*.$$

Finally, (2.13) and (2.14) become respectively

$$u^- = 1, \quad (3.21)$$

$$v(z(\tau), \tau) = \exp \left\{ \frac{\lambda}{RT_0} \left(1 - \frac{1}{u^+} \right) \right\}, \quad \tau > \tau^*. \quad (3.22)$$

At this point, it is natural to choose

$$\sigma = \frac{\varepsilon \lambda \rho_w L^2}{k_1 T_0}, \quad (3.23)$$

so that, setting

$$\theta = \frac{(\rho c)_2 T_0}{\varepsilon \lambda \rho_w}, \quad (3.24)$$

condition (3.20) becomes

$$\frac{dz}{d\tau} = u_\xi^- - \frac{k_2}{k_1} u_\xi^+ - \theta(u^+ - 1) \frac{dz}{d\tau} - \varepsilon \frac{\rho_0 c_v}{(\rho c)_1} \frac{t_1}{t_3} v^+ v_\xi^+ \frac{u^+ - 1}{u^+}, \quad \tau > \tau^*. \quad (3.25)$$

4 A reasonable simplified model

It is immediately checked that, while σ and the characteristic diffusion times t_1 and t_2 are of the same order of magnitude (using the data of [2] we have indeed $\sigma \simeq 325$, $t_1 \simeq 846$, $t_2 \simeq 870$), $t_3 = 0.38$ and thus σ/t_3 is of the order 10^3 .

Therefore, it is reasonable to substitute (3.10) with the much simpler relationship

$$\frac{\partial}{\partial \xi} \left(\frac{v}{u} \frac{\partial v}{\partial \xi} \right) = 0, \quad z(\tau) < \xi < 1, \quad \tau > \tau^*, \quad (4.1)$$

Note that (3.9), (3.19) and (3.25) contain the ratios σ/t_3 , t_2/t_3 , t_1/t_3 , but there are also multiplying factors of the order $\rho_0/\rho_w \sim 10^{-3}$.

Now, we use again symbols x and t instead of ξ and τ and denoting by capital letters A, B, C, \dots the constants $O(1)$ appearing in the equations written in Section 3, we can state the following classical formulation of our problem:

Find a constant t^* , a decreasing function $z(t)$, $z(t^*) = 1$, $z(t) > 0$ and two functions $u(x, t)$, $v(x, t)$ possessing all the regularity we will need and such that:

$$\begin{cases} u_t = Au_{xx}, & \text{in } D_0 = (0, 1) \times (0, t^*), \\ u(x, 0) = \bar{u}, & x \in (0, 1), \\ u_x(0, t) = 0, & t \in (0, t^*), \\ -u_x(1, t) = \Gamma_1[u(1, t) - u_\infty], & t \in (0, t^*), \end{cases} \quad (4.2)$$

where $\bar{u} \in (0, 1)$ and $A = \sigma/t_1$.

$$0 < u(x, t) < 1, \quad \text{in } D_0, \quad (4.3)$$

$$u(1, t^*) = 1. \quad (4.4)$$

$$\begin{cases} u_t = Au_{xx}, & \text{in } D_1 = \{(x, t) : 0 < x < z(t), t > t^*\}, \\ u(x, t^*+) = u(x, t^*-), & x \in (0, 1), \\ u_x(0, t) = 0, & t > t^*, \\ u(z(t), t) = 1, & t > t^*. \end{cases} \quad (4.5)$$

$$0 < u(x, t) < 1, \quad \text{in } D_1, \quad (4.6)$$

$$\begin{cases} u_t = Bu_{xx} + C \left[vv_x \left(1 - \frac{1}{u} \right) \right]_x & \text{in } D_2 = \{(x, t) : z(t) < x < 1, t > t^*\} \\ -u_x(1, t) = \Gamma_2[u(1, t) - u_\infty] & t > t^*, \end{cases} \quad (4.7)$$

$$B = \sigma/t_2, \quad C = \varepsilon\rho_0c_v\sigma/[(\rho c)_2t_3].$$

$$\left(\frac{vv_x}{u} \right)_x = 0, \quad \text{in } D_2, \quad (4.8)$$

$$v(z(t), t) = \exp\{G(1 - 1/u^+)\}, \quad t > t^*, \quad (4.9)$$

$$v(1, t) = 1, \quad t > t^*, \quad (4.10)$$

and

$$z'(t) = Hv^+v_x^+/u^+, \quad t > t^*, \quad (4.11)$$

$$z'(t) = u_x^- - Mu_x^+ - \theta(u^+ - 1)z'(t) - Nv^+v_x^+ \left(\frac{u^+ - 1}{u^+} \right), \quad t > t^*, \quad (4.12)$$

$$1 < u(x, t) < u_\infty, \quad \text{in } D_2, \quad (4.13)$$

$$v(x, t) > 1, \quad \text{in } D_2. \quad (4.14)$$

where $G = \lambda/[RT_0]$, $H = \sigma\rho_0/[t_3\rho_w]$, $M = k_2/k_1$ and $N = \varepsilon\rho_0c_vt_1/[(\rho c)_1t_3]$. Since (4.2) is a standard heat conduction problem we can solve it. Then (4.3) is a consequence of maximum principle and the existence of t^* follows immediately from (4.4) if $u_\infty > 1$.

Let us write, from (4.8)

$$\frac{vv_x}{u} = -f(t), \quad \text{in } D_2, \quad (4.15)$$

for an unknown positive function $f(t)$ and consider the free boundary problem for $u(x, t)$ consisting in (4.5) and

$$u_t = Bu_{xx} - Cf(t)u_x, \quad \text{in } D_2. \quad (4.16)$$

$$-u_x(1, t) = \Gamma_2(u(1, t) - u_\infty), \quad t > t^*, \quad (4.17)$$

$$z'(t) = -Hf(t), \quad t > t^*, \quad (4.18)$$

$$z'(t) = u_x^- - Mu_x^+ - \theta(u^+ - 1)z'(t) + Nf(t)(u^+ - 1), \quad t > t^*, \quad (4.19)$$

with the condition on $f(t)$

$$(v^2)_x = -2f(t)u(x, t), \quad \text{in } D_2, \quad (4.20)$$

i.e.

$$f(t) = -\frac{1 - \exp\{2G(1 - 1/u^+)\}}{2 \int_{z(t)}^1 u(x, t) dx}. \quad (4.21)$$

Thus, for any given $f(t)$, we have transformed the original problem in a free boundary problem (in two "phases") for a single unknown function $u(x, t)$, whose solution in turn gives $f(t)$. The analysis of the well-posedness of this non-standard problem will be performed in a forthcoming paper.

5 Numerical simulation

We show here some numerical simulations where the simplified adimensional model has been used. With reference to the situation sketched in Fig. 1 (where the conditions on the fixed boundaries are shown) we have solved

$$u_t = Au_{xx} \quad \text{in } D_0 \cup D_1 \quad (5.1)$$

$$u_t = Bu_{xx} - Cf(t)u_x \quad \text{in } D_2 \quad (5.2)$$

where $f(t)$ is given by (4.21) and $z(t)$ ($z(t^*) = 1$) is the free boundary on which the following conditions are prescribed

$$u(z(t)^-, t) = 1 \quad (5.3)$$

$$z'(t) = -Hf(t) \quad (5.4)$$

$$z'(t) [1 + \theta(u^+ - 1)] = u_x^- - Mu_x^+ + Nf(t)(u^+ - 1). \quad (5.5)$$

The following values have been taken for the physical quantities (taken from [10] [11]).

ρ_s	1.6	g/cm^3
ρ_v	0.001	g/cm^3
ρ_w	1.0	g/cm^3
c_s	0.74	$cal/(g K)$
c_v	0.48	$cal/(g K)$
c_w	1.0	$cal/(g K)$
k_s	0.00136	$cal/(sec cm K)$
k_w	0.00127	$cal/(sec cm K)$
L	1.0	cm
ε	0.3	
K	$1.0E - 9$	darcy
μ	0.0013	poise
R	0.11	$cal/(g * K)$
γ_1	0.002	$cal/(sec * cm^2 * K)$
γ_2	0.003	$cal/(sec * cm^2 * K)$
T_0	373.16	K
λ	540.2	cal/g
p_0	1013250.0	$g/(cm * sec^2)$

Consequently the constants appearing in (5.1)-(5.5) and (4.21) are the following:

$$\begin{aligned} A &= 0.38, & B &= 0.37 \\ C &= 0.08, \\ H &= 0.49, & \theta &= 1.9 \\ M &= 0.71, & N &= 0.16 \\ G &= 13.16. \end{aligned}$$

The constants appearing in the initial and boundary conditions are

$$\begin{aligned} u_0 &= 0.745 \\ u_\infty &= 1.428 \\ \Gamma_1 &= 1.50 \\ \Gamma_2 &= 3.15 \end{aligned}$$

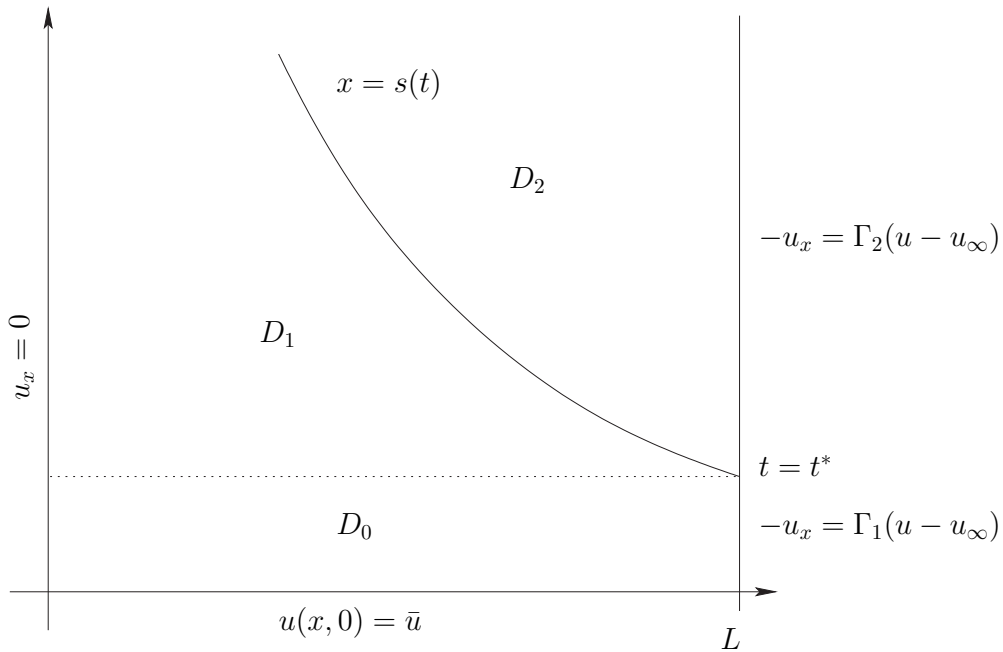


Figure 1: Sketch of the problem

In Figure 2 we show the evolution of the rescaled free boundary as a function of the rescaled time.

Figure 3 gives the rescaled temperature profile in the two zones, at different times.

In Figures 4 and 5 we display the same quantities, but we simulate the crust formation as a relaxation in the heat exchange coefficient Γ_2 . In particular we assumed

$$\Gamma_2(0) = \Gamma_2 \quad (5.6)$$

$$\Gamma_2(t) = \gamma (u(1, t) - 1)_+ \quad (5.7)$$

Acknowledgments. We are indebted to Prof. A. Fasano for many interesting discussions.

References

- [1] A. Bouddour, L. Auriault, M. Mhamdi-Alaoui, *Heat and mass transfer in wet porous media in presence of evaporation-condensation*, J.Heat Mass Transfer **41** (1998) 2263-2277.
- [2] B.E.Farkas, R.P.Singh and T.R.Rumsey, *Modeling heat and mass transfer in immersion frying. I,model development*, J.Food Eng. **29** (1996) 211-226.
- [3] B.E.Farkas, R.P.Singh and T.R.Rumsey, *Modeling heat and mass transfer in immersion frying. II,model solution and verification*, J.Food Eng. **29** (1996) 227-248.

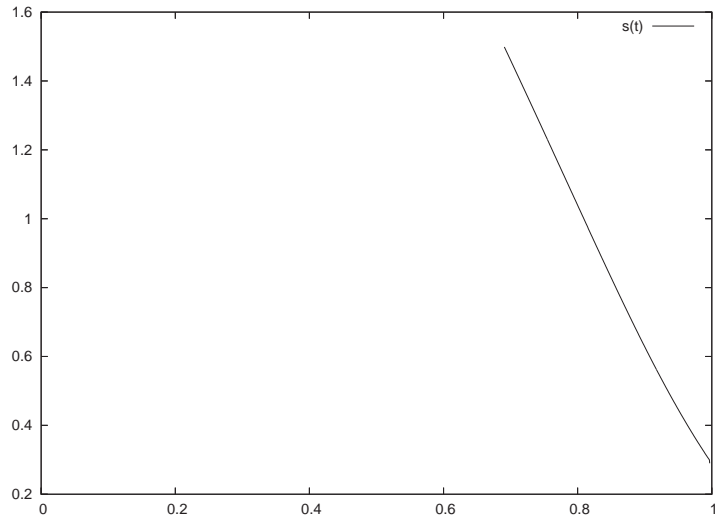


Figure 2: Evolution of the free boundary

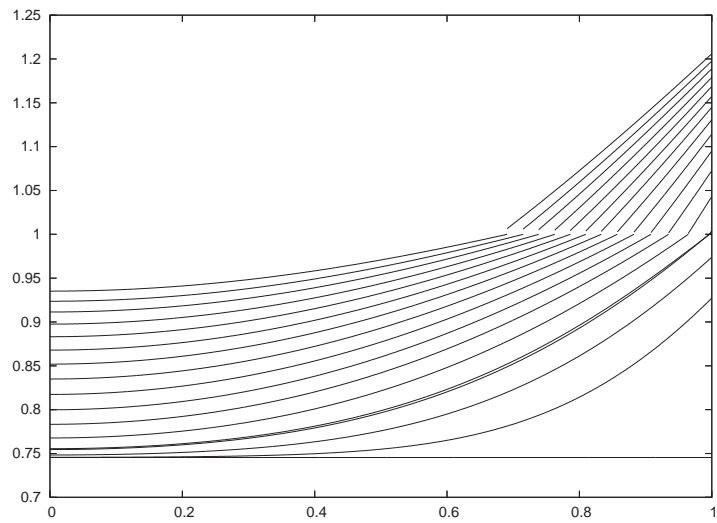


Figure 3: Evolution of temperature profile

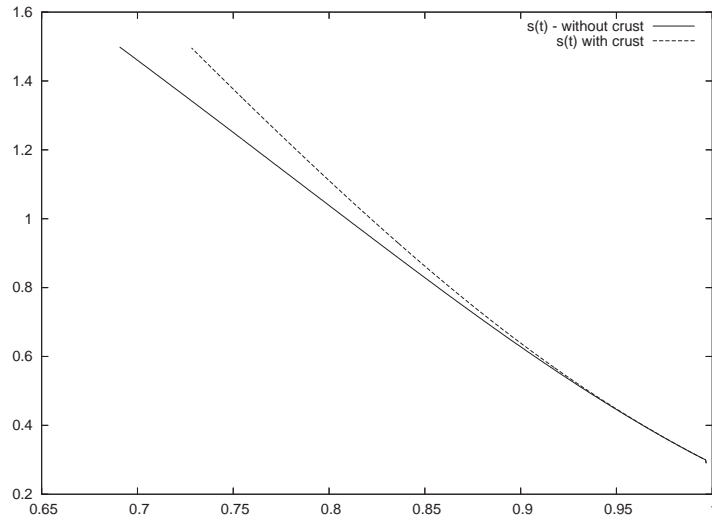


Figure 4: Free boundary with and without “crust formation”

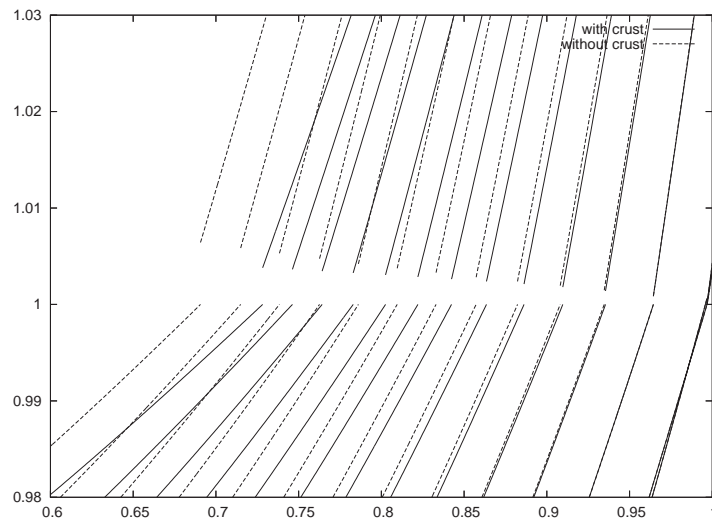


Figure 5: Evolution of temperature with and without “crust formation”

- [4] M. Faryd, *A unified approach to the heat and mass transfer in melting, solidification, frying and different drying processes*, Chem. Eng. Sci. **56** (2001) 5419-5427.
- [5] A. Fasano, A. Mancini, *A mathematical model for a class of frying processes*, International Journal of Computers and Mathematics With Applications ,to appear.
- [6] A. Fasano, A. Mancini, *Modelling Frying of a non deformable specimen by immersion in edible oil*,to appear.
- [7] R. F. Stier, *Chemistry of frying and optimization of deep-fat fried food flavour - An introductory review* , Eur. J. Lipid Sci. Technol. **102** (2000) 507-514.
- [8] O. Vitrac, G. Trystram, A.L. Raoult-Wack, *Deep-fat frying of food: heat and mass transfer, transformations and reactions inside the frying material* , Eur. J. Lipid Sci. Technol. **102** (2000) 507-514.
- [9] R. Yamsaengsung, R.G. Moreira, *Modelling the transport phenomena and structural changes during deep fat frying - Part I: model development* , J. of Food Engineering **53** (2002) 1-10.
- [10] M.W.Zemansky, (1957) *Heat and Thermodynamics*, (4th ed.) Mc Graw-Hill.
- [11] <http://www.thermexcel.com/english/tables/Steam>, vapour, density, latent, sensible heat vapourization, specific, volume, pressure, dynamic viscosity, enthalpy.htm.