Coal–Water Slurry technology: problems and modeling solutions

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Firenze - 29 october 2002
Characteristics of the product

Mixture of coal (up to 70% in weight), water (up to 29%) and suitable fluidizing agents (about 1%). Coal particles are micronized with a top size of about 250 $\mu m$ and a bimodal size distribution centered at 10 and 100 $\mu m$ for optimal (maximum) packing.
Industrial problems

The product is totally stable at rest (therefore it can be stocked for long periods of time) and burned without needing a preliminary dehydration. There are however two main problems:

- Rheological degradation
- Sedimentation
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- **Sedimentation**
Industrial problems

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- **Rheological degradation**: it’s a long-time effect due to shear. The apparent viscosity reaches extremely high values and the product becomes useless.

- **Sedimentation**: it’s also a long-time effect due to manufacturing impurities which are not stabilized by chemical surfactant. A sedimentation bed grows up on the bottom of the first kilometers of a pipeline eventually compromising the optimal discharge.
Degradation

Relative apparent viscosity at $10 \text{ s}^{-1}$ vs. specific cumulative energy (kJ/kg) for a polish CWS. The different marks identify mixtures with 0.5%, 0.75%, and 1.00% of dispersed additive.
Degradation

Relative apparent viscosity at $10 \, s^{-1}$ vs. time for two different CWS at various shear rates.

The white and green marks identify a type of mixture (Colombian CWS) at two different shear rates ($20 \, s^{-1}$ and $50 \, s^{-1}$); the other marks identify another mixture (Russian CWS) at three different shear rates ($20 \, s^{-1}$, $50 \, s^{-1}$, and $80 \, s^{-1}$).
Degradation

Relative apparent viscosity at 10 $s^{-1}$ vs. specific cumulative energy using the same data of the previous plot. All marks related to the same type of mixture arrange themselves on a unique curve *regardless of the operated shear rate*.
Additive dynamics

Main variables: $A$ % of additive available in water, $B$ % of additive adsorbed by non-ionized sites on coal particles, $Y$ concentration of ions adsorbed on coal particles, $I$ concentration of ions in water, $\bar{B}$ maximum quantity of dispersant adsorbable on coal particles, $D$ % of “inert” additive adsorbed on coal particles
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Main facts:

- Internal frictions cause the transition $B \rightarrow D$ and $I \rightarrow Y$.
- The transition $A \rightarrow B$ occurs to replace the dispersant becoming inert.
- While $A \rightarrow B$ is reversible, $I \rightarrow Y$ and $B \rightarrow D$ are not.
- Irreversible transitions are activated only by internal dissipation due to shear.
Additive dynamics

\[ \begin{align*}
\dot{A} &= -\mu_1 A(\bar{B} - B) + \mu_2 B \\
\dot{B} &= \mu_1 A(\bar{B} - B) - \mu_2 B \\
\dot{D} &= -\lambda Y D \\
\dot{Y} &= \alpha_1 (\bar{B} - B - Y)(I_0 - Y) - \alpha_2 Y \\
\dot{\bar{B}} &= f(W)(B_\infty - \bar{B})
\end{align*} \]

with initial conditions \( A(0) = A_0, B(0) = B_0, I(0) = I_0, f(W) \) function of the dissipated power and \( B_\infty \) asymptotic value of \( B \). Constants \( \mu_1, \mu_2 \) are the rate of adsorption and desorption respectively. Clearly \( I(t) + Y(t) = I_0 \) and \( A + B = D \).
Degradation in a pilot loop

Once $B$ is determined, the CWS mixture can be modeled as a *Bingham fluid* where the characteristic rheological parameters are functions of $B$.

Experimental data in a batch reactor fit very well the model (in this case all parameters depend only on time, not on spatial coordinates).

However in a pipeline the spatial dependence of rheological parameters cannot be neglected and the problem is much more complicated!
Degradation in a pilot loop

Quasi–steady approximation: the degradation time scale is much larger than the loop circulation time

Main variables in the axisymmetric geometry: \( \tau(r, t) \) shear stress, \( \eta_B \)
(constant) plastic viscosity, \( \tau_0(r, t) \) yield stress, \( v(r, t) \) velocity, \( r \) radius,
\( G(t) \) pressure gradient, \( s(t) \) free surface
Degradation in a pilot loop

\[
(\tau - \tau_0)^+ = \eta_B |\partial_r v|, \quad r \in (0, R), t > 0
\]

\[
G(t) = \frac{1}{r} \partial_r (r \tau) \quad r \in (0, R), t > 0
\]

\[
\partial_r v \big|_{r=s(t)} = 0 \quad t > 0
\]

\[
s(t) = 2\tau_0 (s(t), t)/G(t) \quad t > 0
\]

\[
\partial_t \tau_0 = \tau |\partial_r v|, \quad r \in (0, 1), t > 0
\]

**Unexpected phenomena:** the free boundary (separating the sheared and the unsheared regions) can touch the pipe wall or a new free boundary can grow on the wall and meet the original one in a finite time. Both cases lead to the blockage of the pipeline.
Sedimentation velocity

The classical Stokes law is doesn’t hold any longer and is replaced by

$$v_s(\dot{\gamma}, r) = \alpha(\dot{\gamma}) r^2$$

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Behaviour of the sedimentation coefficient \( \alpha \) versus shear in a Bingham fluid. The proportionality coefficient of the “square radius”–law depends on the shear rate. Note that \( v_0(0) = 0 \)!
Sedimentation dynamics in a pipe

Model variables: $r$ pipe radial coordinate, $x$ longitudinal axis of the pipe, $t$ time, $\dot{\gamma}(r)$ (known function of $\tau, \tau_0, \eta_B$)
Sedimentation dynamics in a pipe

Simplifying assumptions: the ratio “sediment thickness/pipe radius” is rather small, rheological parameters remain constant (the degradation time scale is much greater than the pipelining time), the sediment bed is partially transported away by the main flux, the geometry of the sediment cross-section is “essentially known”, the mass flux of sediment per unit time $q(x, t)$ through a cross-section at distance $x$ is a known function of $a(x, t)$, the area of the sediment cross-section at distance $x$ and time $t$. 
Consequences: the total sediment rate \( S_T(x) \) per unit length of the pipe at distance \( x \) from the origin can be explicitly evaluated as a function of the settling particle concentration and the convective velocity of the main flux.
Sedimentation dynamics in a pipe

Evolution equation for $a(x, t)$

$$\frac{\partial a}{\partial t} + q'(a) \frac{\partial a}{\partial x} = S_T(x)$$

with the initial–boundary conditions $a(x, 0) = a(0, t) = 0$
Solutions

- best regime:
- attention regime:
Solutions

- **best regime:** if the maximum thickness of sediments remains lower than a threshold $\Delta$ (which depends on the discharge) then $q = \lambda a$ (trivial case) $\Rightarrow$ sediments never sticks to the bottom of the pipeline

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In all cases of physical interest the solution exists globally
Sedimentation bed profiles

Bed profile at various time: behind the maximum the bed remains stationary

dynamics of the bed
Sedimentation bed profiles

<table>
<thead>
<tr>
<th>$Q \ (m^3/h)$</th>
<th>100</th>
<th>150</th>
<th>250</th>
<th>450</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \ (cm)$</td>
<td>0.7</td>
<td>1.0</td>
<td>1.8</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Estimated values of $\Delta$ as a function of $Q$
Sedimentation bed profiles

<table>
<thead>
<tr>
<th>( Q \ (m^3/h) )</th>
<th>( \Delta \ (cm) )</th>
<th>Gap (cm)</th>
<th>( t_{cr} \ (days) )</th>
<th>( x_{cr} \ (Km) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.7</td>
<td>3.5</td>
<td>0.4</td>
<td>0.48</td>
</tr>
<tr>
<td>150</td>
<td>1.0</td>
<td>4.2</td>
<td>0.5</td>
<td>1.24</td>
</tr>
<tr>
<td>250</td>
<td>1.8</td>
<td>5.4</td>
<td>( \infty )</td>
<td>( \geq 100 )</td>
</tr>
<tr>
<td>450</td>
<td>3.2</td>
<td>7.1</td>
<td>( \infty )</td>
<td>( \geq 100 )</td>
</tr>
</tbody>
</table>

Critical time and distances needed by the static sediment to reach 2\% (=1cm) of the pipe diameter

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**Sedimentation bed profiles**

<table>
<thead>
<tr>
<th>$\Delta$ (cm)</th>
<th>$t_{cr}$ (days)</th>
<th>$x_{cr}$ (Km)</th>
<th>$T_{cr}$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.71</td>
<td>3.57</td>
<td>1.01</td>
</tr>
<tr>
<td>1.5</td>
<td>0.97</td>
<td>4.99</td>
<td>1.35</td>
</tr>
<tr>
<td>1.6</td>
<td>1.51</td>
<td>7.5</td>
<td>2.08</td>
</tr>
<tr>
<td>1.7</td>
<td>3.20</td>
<td>14.2</td>
<td>3.85</td>
</tr>
<tr>
<td>1.8</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>1.9</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>2.0</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Critical times and distances: $T_{cr}$ is the time needed by the system to reach a state in which the thickness of the static layer is above $h_{cr} = 2R/100$ over a longitudinal section of the pipe with length $\approx L/100$. 

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pipe radius $R = 25\, cm$, discharge $Q = 250 \, m^3/h$
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