The MAC-GEO project

MAthematical modelling for government control of public Concession (licence) for exploitation of GEOthermal resources

Scientific coordinator: Fabio Rosso
Dept. Mathematics Ulisse Dini, Univ. Firenze, Italy

Argentina, september 2010

People involved

Research group in Florence: I. Borsi, M. Ceseri, A. Farina, A. Fasano, M. Primicerio, A. Speranza

Students: M. Cerminara, L. Meacci

Institutions involved (at UNIFI): Dept. of Math. Ulisse Dini, I2T3, Dept. Earth Sciences, Dept. Informatics Systems, Media Integration and Communication Center

Institutions involved (at UNIBO): Dept. of Chem. Engineering, Mines and Environmental Technologies

Institutions involved (at C.N.R.): Institute of Bio-Meteorology, National Interuniversitary Consortium for the Engineering of Geo-resources

total funds: 790,000 €

Aims of the project

Aims of the project

- To setup a “full field” mathematical model for reasonable and responsible predictions about the long-time behavior of **geothermal reservoirs** in Tuscany (Italy) under standard industrial energy production regimes
- To check the environmental impact of deep geothermal fluids extraction process upon phreatic superficial water layers
- The final product should be an easy-to-use package for people in government supervising structures (because energy resources remain a state property also if a private society carry on the necessary technology)
- The underlying numerical code must interface with a G.I.S. database carrying all field historical data (geological, extraction, production and so on) to generate up-to-date reliable simulations

Aims of the project

- To setup a "full field" mathematical model for reasonable and responsible predictions about the long-time behavior of **geothermal reservoirs** in Tuscany (Italy) under standard industrial energy production regimes
- To check the environmental impact of deep geothermal fluids extraction process upon phreatic superficial water layers
- The final product should be an easy-to-use package for people in government supervising structures (because energy resources remain a state property also if a private society carry on the necessary technology)
- The underlying numerical code must interface with a G.I.S. database carrying all field historical data (geological, extraction, production and so on) to generate up-to-date reliable simulations

Aims of the project

- To setup a "full field" mathematical model for reasonable and responsible predictions about the long-time behavior of **geothermal reservoirs** in Tuscany (Italy) under standard industrial energy production regimes
- To check the environmental impact of deep geothermal fluids extraction process upon phreatic superficial water layers
- The final product should be an easy-to-use package for people in government supervising structures (because energy resources remain a state property also if a private society carry on the necessary technology)
- The underlying numerical code must interface with a G.I.S. database carrying all field historical data (geological, extraction, production and so on) to generate up-to-date reliable simulations

Aims of the project

- To setup a "full field" mathematical model for reasonable and responsible predictions about the long-time behavior of **geothermal reservoirs** in Tuscany (Italy) under standard industrial energy production regimes
- To check the environmental impact of deep geothermal fluids extraction process upon phreatic superficial water layers
- The final product should be an easy-to-use package for people in government supervising structures (because energy resources remain a state property also if a private society carry on the necessary technology)
- The underlying numerical code must interface with a G.I.S. database carrying all field historical data (geological, extraction, production and so on) to generate up-to-date reliable simulations

Is this an easy task?

Is this an easy task?

No, it's not!

Is this an easy task?

No, it's not!
A geothermal reservoir is a very complex physical system in which chemistry, geology, thermodynamics and the physics of fluids in porous-fractured media are all combined in a highly nonlinear way.

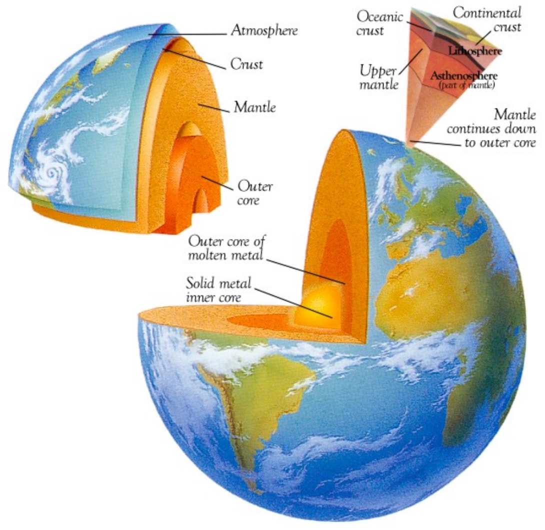
Is this an easy task?

No, it's not!
A geothermal reservoir is a very complex physical system in which chemistry, geology, thermodynamics and the physics of fluids in porous-fractured media are all combined in a highly nonlinear way.
Furthermore data access (from 1 to 4 km deep in the Earth crust) is very poor.

Is this an easy task?

No, it's not!
A geothermal reservoir is a very complex physical system in which chemistry, geology, thermodynamics and the physics of fluids in porous-fractured media are all combined in a highly nonlinear way.
Furthermore data access (from 1 to 4 km deep in the Earth crust) is very poor.
Let's take a look to geothermal energy from a general point of view.

Earth internal structure



Heat flux sources

Total heat power of the earth is evaluated to be 42 TW (4.2×10^{13} J/s). Where does all this heat come from?

Heat flux sources

Total heat power of the earth is evaluated to be 42 TW (4.2×10^{13} J/s). Where does all this heat come from?

There are several sources of heat inside Earth.

Heat flux sources

Total heat power of the earth is evaluated to be 42 TW (4.2×10^{13} J/s). Where does all this heat come from?

There are several sources of heat inside Earth.

The decay of naturally radioactive elements within the mantle (Potassium-40, Uranium-238, Uranium-235, and Thorium-232 are the most important) is commonly believed to be the leading source (up to 80 % of total).

Heat flux sources

Total heat power of the earth is evaluated to be 42 TW (4.2×10^{13} J/s). Where does all this heat come from?

There are several sources of heat inside Earth.

The decay of naturally radioactive elements within the mantle (Potassium-40, Uranium-238, Uranium-235, and Thorium-232 are the most important) is commonly believed to be the leading source (up to 80 % of total).

For example Thorium-232 has a half-live time of 14.1×10^9 years Uranium-238 of 4.46×10^9 years.

Heat flux sources

Total heat power of the earth is evaluated to be 42 TW (4.2×10^{13} J/s). Where does all this heat come from?

There are several sources of heat inside Earth.

The decay of naturally radioactive elements within the mantle (Potassium-40, Uranium-238, Uranium-235, and Thorium-232 are the most important) is commonly believed to be the leading source (up to 80 % of total).

For example Thorium-232 has a half-live time of 14.1×10^9 years Uranium-238 of 4.46×10^9 years.

Since the Earth is estimated to be 4.6×10^9 years old these nuclei have not had time to completely decay away since the formation of the Earth.

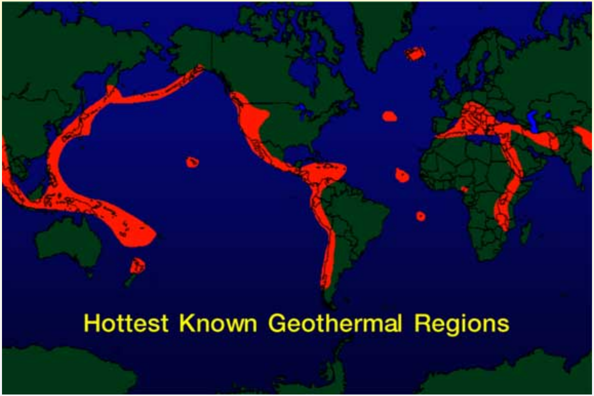
The heat flux is not the same everywhere!

The heat flux is not the same everywhere!

This depends on the fact that along tectonic plates boundaries and subduction zones, the Earth crust is thinner.

The heat flux is not the same everywhere!

This depends on the fact that along tectonic plates boundaries and subduction zones, the Earth crust is thinner.



Geothermal resource (from Greek Gêo =Earth and Thermòs=heat)

The Earth heat power cannot be used directly: water is the "carrier"medium. A geothermal resource is a natural underground basin rich of overheated fluids (water) which can be extracted to economic or social purposes.

(a) Geysers, Atacama desert

(b) Geysers, Island

Geothermal gradient

The standard geothermal gradient is about $2 \div 3\ {}^{\circ}C/100m$.

An economically significant geothermal gradient should be $\geq 7\ {}^{\circ}C/100m$.

Close to the Earth surface geothermal energy is sufficient to bring water to the boiling point. Then vapor strength can move power generators.

This kind of energy is renewable (essentially infinite on the human time scale).

Some history

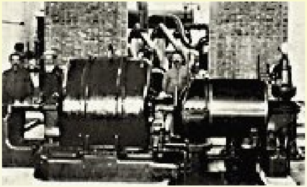
Geothermal energy **at low enthalpy** (thermal springs) has been known for centuries and used by animals and humans.

Some history

Geothermal energy **at low enthalpy** (thermal springs) has been known for centuries and used by animals and humans.
Geothermal energy **at high enthalpy** is relatively recent:
4 July 1904 Prince Piero Ginori Conti (son in law of Earl Florestano de Larderel) tested the first geothermal power generator at the Larderello dry steam field in Italy

Some history

1911: the world's first geothermal power plant (250 KW) was built in the Devil's Valley of Larderello.



This remained the world's only industrial producer of geothermal energy until 1958 when New Zealand built a plant of its own in Wirakei

Actually there geothermal power plants in 24 countries

Geothermal power in the world

Geothermal power in the world

Power generation (24 countries worldwide) through high enthalpy plants: more than 10 GW (power capacity).

Geothermal power in the world

Power generation (24 countries worldwide) through high enthalpy plants: more than 10 GW (power capacity).

Direct use (heating, thermal springs, enhanced growing,...): more than 28 GW

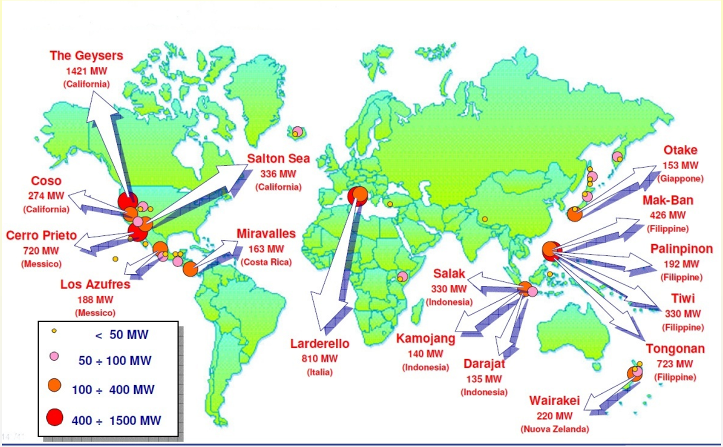
Geothermal power in the world

Power generation (24 countries worldwide) through high enthalpy plants: more than 10 GW (power capacity).

Direct use (heating, thermal springs, enhanced growing,...): more than 28 GW

Reduced environmental impact: a reduction of 118×10^6 tons of CO_2 per year of athmosferic pollution

Main geothermal basins around the World



Geothermal energy production

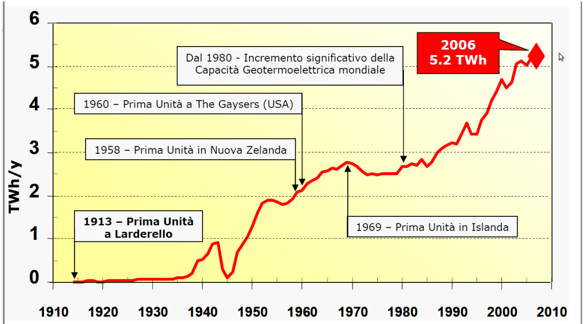
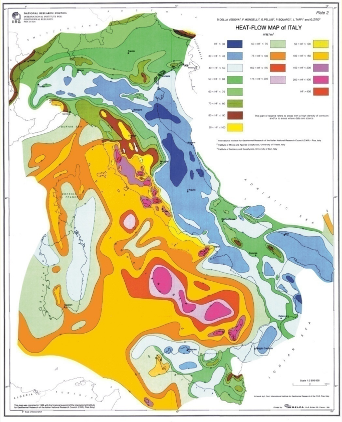
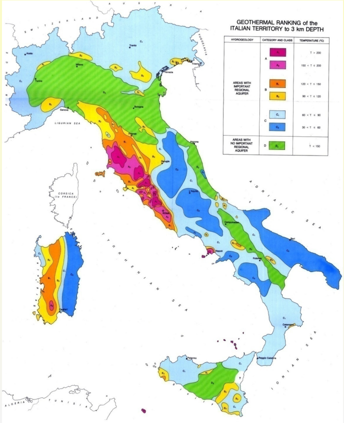


Figura 2 – Incremento della produzione geotermoelettrica in Italia e realizzazioni impiantistiche geotermiche in altri paesi.

Heat flux in Italy

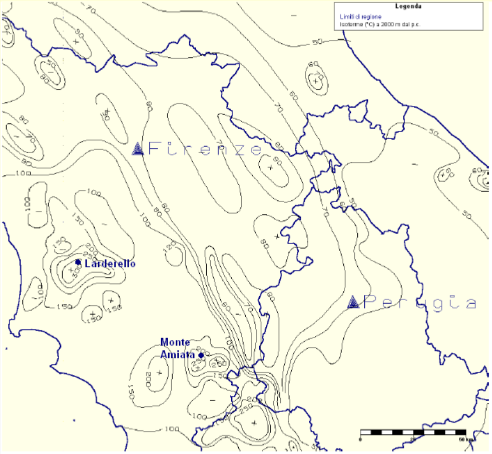


(c)

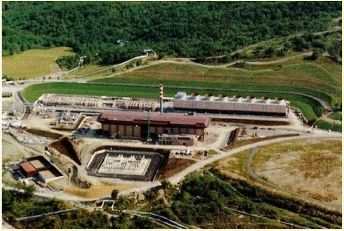


(d)

Interested areas in Tuscany



Geothermal activity in Tuscany



(e) Power generation by geothermal energy (high enthalpy) covers more than 28 % of the regional necessities (2 % of the national ones) with ≈ 800 MW capacity

(f) Low enthalpy production is also important for human needs (house heating, fish and vegetable product artificial growing-up, ...)

How it works

How it works

- Magmatic intrusion is the deep (but not too deep) heat source. To exploit this energy source we have to be able to bring heat to the surface.

How it works

- Magmatic intrusion is the deep (but not too deep) heat source. To exploit this energy source we have to be able to bring heat to the surface.
- Above the heat source is the reservoir: fractured-porous rocks where fluids (liquid/vapor) at high temperature and pressure may circulate.

How it works

- Magmatic intrusion is the deep (but not too deep) heat source. To exploit this energy source we have to be able to bring heat to the surface.
- Above the heat source is the reservoir: fractured-porous rocks where fluids (liquid/vapor) at high temperature and pressure may circulate.
- The reservoir need to be capped by an almost impermeable layer (clay) to prevent fluids and heat losses.

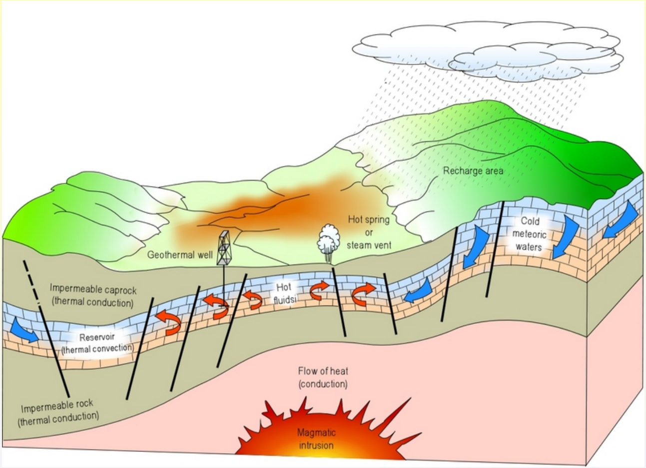
How it works

- Magmatic intrusion is the deep (but not too deep) heat source. To exploit this energy source we have to be able to bring heat to the surface.
- Above the heat source is the reservoir: fractured-porous rocks where fluids (liquid/vapor) at high temperature and pressure may circulate.
- The reservoir need to be capped by an almost impermeable layer (clay) to prevent fluids and heat losses.
- The systems behaves like a natural “caldera”(pressure boiling pan)

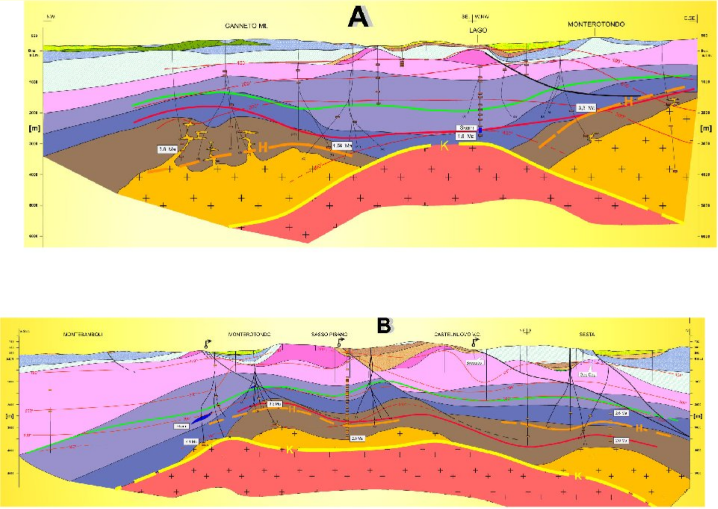
How it works

- Magmatic intrusion is the deep (but not too deep) heat source. To exploit this energy source we have to be able to bring heat to the surface.
- Above the heat source is the reservoir: fractured-porous rocks where fluids (liquid/vapor) at high temperature and pressure may circulate.
- The reservoir need to be capped by an almost impermeable layer (clay) to prevent fluids and heat losses.
- The systems behaves like a natural “caldera”(pressure boiling pan)
- If exploited, the basin needs to be recharged either by meteoric waters (rain) or artificial re-injection

General structure of a geothermal basin



Larderello basin cross-sections



Physical characteristics

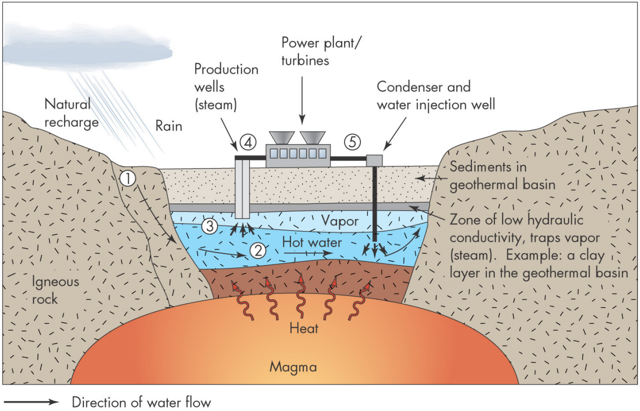
The Larderello-Travale reservoir produces thermal fluids at high enthalpy ($T = 150 \div 260°C$ and $P = 2 \div 15$ bar). These fluids are largely overheated vapor and minor quantities (15 % in wt.) of gas (essentially CO_2) In the deep reservoir $T_{\max} \approx 350°C$ and $P_{\max}$ (of vapor) ≈ 70 bar. This is a typical *vapor dominated basin.*

Unlike Larderello-Travale, the Mount Amiata reservoir is a *water dominated* basin. At top well geothermal fluids are two-phase mixtures (water and vapor) at $P \approx 20$ bar e $T \approx 130° \div 190°C$ with very high salinity ($10 \div 12$ g/l) In thew deep reservoir the (hydrostatic) pressure $P \approx 200 \div 250$ bar and $T \approx 300 \div 360°C$

Geometry

- Depth: 10 Km
- Width:
  - Larderello: 50 Km
  - Amiata: 40 Km
- Length:
  - Larderello: 60 Km
  - Amiata: 50 Km

"Vapour dominated" schematic picture



1. Natural recharge of water from rain
2. Hot water produced by earth processes
3. Steam to production well
4. Steam to turbines to produce electricity
5. Water is injected back into ground

Thermodynamics

Crucial hypothesis: local thermodynamical equilibrium. This assumption is assumed also near any extraction well.

Thermal equilibrium is of course not true on the full scale of the reservoir (temperature and pressure vary significantly over the entire geothermal basin).

Available geophysical data

Deep rocks data like permeability and porosity are scarcely available. This situation suggests two possible modeling scenarios

1. *Continuum-equivalent model*, that is:
   (a) Constant porosity
   (b) Isotropic constant absolute permeability, that is the permeability tensor is

$$\mathbf{K} = K\,\mathbf{Id},$$

   with K constant and $\mathbf{Id}$ identity matrix.
2. *Double permeability and porosity model* (in progress)

The former is obviously easier but the latter is closer to reality.

Multiphase multicomponent model

ϕ	porosity
$i = 1, ..., N$	component index
$\alpha = l, g$	phase index
X_i^α	mass fraction
ρ^α	absolute α–phase density
$\rho_i^\alpha = \rho^\alpha X_i^\alpha$	i–component density in phase α
S^α	α–phase saturation
$\rho_i^\alpha S^\alpha \phi$	i–component density in phase α **in the porous medium**
P^l	partial pressure of liquid phase
P^g	partial pressure of gas phase

The geothermal fluid is a mixture of H_2O in liquid and vapor phase with a *non negligible* presence of *Non-Condensible Gases* (essentially CO_2). These gases may be dissolved in the liquid phase.

Physical hypotheses

1. Rocks are the porous matrix that hosts the geothermal fluid which is a mixture of
   - a liquid phase (l);
   - a gas phase (g).

   therefore

   $$1 = \sum_{i=1}^{N} X_i^{\alpha}, \text{ per } \alpha = l, g. \tag{1}$$

2. the porous medium is saturated, that is it does not host *dry air*: this means

   $$S^l + S^g = 1. \tag{2}$$

3. the two phases are in thermodynamical equilibrium
4. The fluid flux is due only to convection (**diffusivity is negligible**)

Mass and energy balance

For each component i in phase α we write

$$\frac{\partial}{\partial t}\left(\rho^\alpha X_i^\alpha S^\alpha \phi\right)+\nabla\cdot\left(\rho^\alpha X_i^\alpha S^\alpha \phi \mathbf{v}_i^\alpha\right) = \frac{M_i^\alpha}{M_{tot}}\frac{1}{V_{ext}}\Psi^{ext}+\left(\rho^\alpha X_i^\alpha S^\alpha \phi\right)\Gamma^\alpha, \tag{3}$$

with

- $\mathbf{v}_i^\alpha$ is the velocity of component i in phase α.
- Ψ^{ext} is the **total** mass of fluid extracted (or injected) in the unit time.
- V_{ext} is the total volume of the extraction or injection zone
- Γ^α is the source/sink term due to the change of phase

For a fixed component i, we sum over phases $\alpha = l, g$ equations (3):

$$\sum_{\alpha=l,g} \left(\rho^\alpha X_i^\alpha S^\alpha \phi\right) \Gamma^\alpha = 0,$$

Define the ***parent density*** to get

$$\rho_i^{(0)} = \sum_{\alpha=l,g} \rho^\alpha X_i^\alpha S^\alpha, \tag{4}$$

$$\frac{\partial}{\partial t}\left(\rho_i^{(0)}\phi\right) + \nabla \cdot \left(\sum_{\alpha=l,g} \rho^\alpha X_i^\alpha S^\alpha \phi \mathbf{v}_i^\alpha\right) = \frac{\rho_i^{(0)}}{\sum_j \rho_j^{(0)}} \frac{1}{V_{ext}} \Psi^{ext}. \tag{5}$$

Some basic assumptions

Some basic assumptions

1. There are **only two** pressures : P^l, pressure of phase l, P^g, pressure of phase g.

Some basic assumptions

1. There are **only two** pressures : P^l, pressure of phase l, P^g, pressure of phase g.
2. The two pressures are related through the *capillary pressure*:

$$P_c = P_c(S^l) = P^g - P^l \tag{6}$$

Some basic assumptions

1. There are **only two** pressures : P^l, pressure of phase l, P^g, pressure of phase g.
2. The two pressures are related through the *capillary pressure*:
$$P_c = P_c(S^l) = P^g - P^l \tag{6}$$
3. Specific discharge of phase α is given by Darcy's law: $\mathbf{q}^\alpha = \phi S^\alpha \mathbf{v}^\alpha = -\mathbf{K}\frac{k_{r\alpha}}{\mu^\alpha}\left(\nabla P^\alpha + \rho^\alpha \mathbf{g}\right)$, where $\mathbf{K}$, is the tensor of absolute permeability of the medium, $k_{rel\alpha}$ the relative permeability of phase α and μ^α is its viscosity.

Some basic assumptions

1. There are **only two** pressures : P^l, pressure of phase l, P^g, pressure of phase g.
2. The two pressures are related through the *capillary pressure*:
$$P_c = P_c(S^l) = P^g - P^l \tag{6}$$
3. Specific discharge of phase α is given by Darcy's law: $\mathbf{q}^\alpha = \phi S^\alpha \mathbf{v}^\alpha = -\mathbf{K}\frac{k_{r\alpha}}{\mu^\alpha}\left(\nabla P^\alpha + \rho^\alpha \mathbf{g}\right)$, where $\mathbf{K}$, is the tensor of absolute permeability of the medium, $k_{rel\alpha}$ the relative permeability of phase α and μ^α is its viscosity.
4. in each phase all components move with the same velocity
$$\mathbf{v}_i^\alpha = \mathbf{v}^\alpha, \ \textbf{for all } i = 1, ..N$$

Some basic assumptions

1. There are **only two** pressures : P^l, pressure of phase l, P^g, pressure of phase g.
2. The two pressures are related through the *capillary pressure*:
$$P_c = P_c(S^l) = P^g - P^l \tag{6}$$
3. Specific discharge of phase α is given by Darcy's law: $\mathbf{q}^\alpha = \phi S^\alpha \mathbf{v}^\alpha = -\mathbf{K}\frac{k_{r\alpha}}{\mu^\alpha}\left(\nabla P^\alpha + \rho^\alpha \mathbf{g}\right)$, where $\mathbf{K}$, is the tensor of absolute permeability of the medium, $k_{rel\alpha}$ the relative permeability of phase α and μ^α is its viscosity.
4. in each phase all components move with the same velocity
$$\mathbf{v}_i^\alpha = \mathbf{v}^\alpha, \ \textbf{for all} \ i = 1, ..N$$
5. P^α and S^α are related via a **constitutive relationship**

Mass and energy balance equation for the ith–component

The above assumptions imply

$$\frac{\partial}{\partial t}\left(\rho_i^{(0)}\phi\right)-\nabla\cdot\left[\sum_{\alpha=l,g}\rho^\alpha X_i^\alpha \mathbf{K}\frac{k_{r\alpha}}{\mu_\alpha}\left(\nabla P^\alpha+\rho^\alpha\mathbf{g}\right)\right]=\frac{\rho_i^{(0)}}{\sum_j \rho_j^{(0)}}\frac{1}{V_{ext}}\Psi^{ext}. \tag{7}$$

$$\frac{\partial}{\partial t}\left[(1-\phi)\rho_r c_r T+\phi\sum_\alpha \rho^\alpha S^\alpha u^\alpha\right]+\sum_\alpha \nabla\cdot(h^\alpha\mathbf{q}^\alpha)=\nabla\cdot[\lambda_{mix}\nabla T], \tag{8}$$

where

$$\lambda_{mix}=(1-\phi)\lambda_r+\phi\sum_\alpha \lambda_\alpha S^\alpha \qquad h^\alpha=\sum_i X_i^\alpha h_i^\alpha$$

u is the internal energy per unit mass and h the enthalpy.

Closure of the system

1. **number of unknowns**: T, P^α, $\rho_i^{(0)}$, ρ^α, S^α, X_i^α, with $\alpha = l, g$ and $i = 1, .., N$. Thus $(3N+7)$ unknowns.

The total number of available eqs. is $(2N+6)$.

We still have $(N+1)$ eqs. missing: these are provided by the thermodynamics (phase equilibrium conditions)

Closure of the system

1. **number of unknowns**: T, P^α, $\rho_i^{(0)}$, ρ^α, S^α, X_i^α, with $\alpha = l, g$ and $i = 1, .., N$. Thus $(3N + 7)$ unknowns.
2. **number of eqs**: actually we have only N eqs (mass balance, 7) plus the energy balance (8). Thus $(N + 1)$ eqs.

The total number of available eqs. is $(2N + 6)$.

We still have $(N + 1)$ eqs. missing: these are provided by the thermodynamics (phase equilibrium conditions)

Closure of the system

1. **number of unknowns**: T, P^α, $\rho_i^{(0)}$, ρ^α, S^α, X_i^α, with $\alpha = l, g$ and $i = 1, .., N$. Thus $(3N + 7)$ unknowns.
2. **number of eqs**: actually we have only N eqs (mass balance, 7) plus the energy balance (8). Thus $(N + 1)$ eqs.
3. **constraints and relations**: that for mass fractions (eqs. 1) and that for saturations (eq. 2). Additionally there is the capillary relation (eq. 6) plus the parent density definition (eq. 4) which relates density and saturations.

The total number of available eqs. is $(2N + 6)$.

We still have $(N + 1)$ eqs. missing: these are provided by the thermodynamics (phase equilibrium conditions)

Phase equilibrium eqs.

We need a general EOS which, by means of the *compressibility factor*,

$$Z^\alpha = \frac{P^\alpha v^\alpha}{RT}, \tag{9}$$

writes

$$\mathcal{F}(Z^g) = 0, \tag{10}$$

where v^α is the molar volume (basically $1/\rho^\alpha$), and $\mathcal{F}$ is a known (generally cubic) function.

Then, following Helmholtz, we define the *chemical potential*

$$\mu_i = \frac{\partial}{\partial \rho_i} F(\rho_i, T),$$

F being the Helmholtz free energy, and introduce the *Gibbs-Duhem eq.*:

$$-P^\alpha = F - \sum_i \rho_i^\alpha \mu_i^\alpha .$$

By definition, phase equilibrium holds in this case (liquid-gas) if the following set of algebraic eqs. is satisfied

$$\mu_i^l = \mu_i^g, \qquad (i = 1, .., N) \tag{11}$$

Eq. 10 and eq. 11 provide the set of $N + 1$ eqs. missing.

Boundary conditions

At the basin bottom: *mass flux equal to zero* and *given temperature* (with constant gradient inside the basin)

At the basin top: depend on the choice of the geometry (if we include or not the superficial (very low permeable) layer.

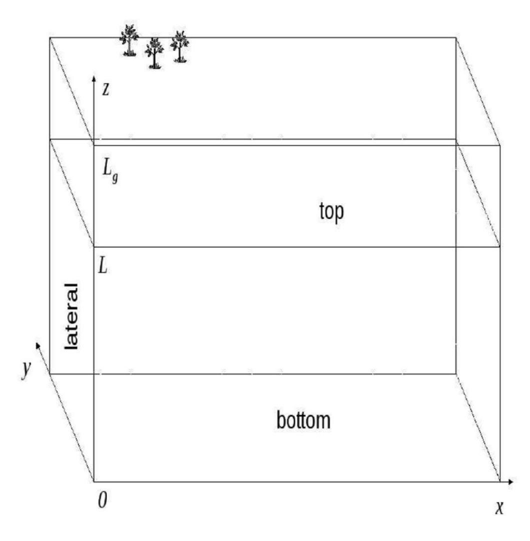
if included: atmospheric pressure and standard temperature

if not included: mass flux equal to zero and given temperature

(possibly a mixture of these two situations)

Lateral conditions: the physical boundary of the basin is really uncertain. Thus we imply *given temperature* (from standard geothermal gradient) and *hydrostatic pressure*

Example: a very simplified geometry



Simplified boundary conditions at $z = 0$

$$T\big|_{z=0} = T_b, \quad \text{with} \quad T_b \quad \text{uniform.}$$

$$\mathbf{q}_l\big|_{z=0} \cdot \mathbf{e}_z = 0, \qquad \Leftrightarrow \qquad \left.\frac{\partial P_l}{\partial z}\right|_{z=0} = -\rho_l g,$$

$$\mathbf{q}_g\big|_{z=0} \cdot \mathbf{e}_z = 0, \qquad \Leftrightarrow \qquad \left.\frac{\partial P_g}{\partial z}\right|_{z=0} = -\rho_g g.$$

Summarized boundary conditions at $z = L$

$$T\big|_{z=L} = T_{top}, \quad \text{with} \quad T_{top} \quad \text{uniform.}$$

$(T_{top} < T_b)$

$$\mathbf{q}_l\big|_{z=L} \cdot \mathbf{e}_z = 0, \qquad \Leftrightarrow \qquad \left.\frac{\partial P_l}{\partial z}\right|_{z=L} = -\rho_l g,$$

$$\mathbf{q}_g\big|_{z=L} \cdot \mathbf{e}_z = 0, \qquad \Leftrightarrow \qquad \left.\frac{\partial P_g}{\partial z}\right|_{z=L} = -\rho_g g.$$

Summarized boundary conditions

At Γ_{lat} (lateral boundary)

$$T|_{\Gamma_{lat}} = T_b - \frac{T_b - T_{top}}{L} z.$$

$P|_{\Gamma_{lat}} = P(z)$, with $P(z)$ solution in $[0, L]$ of the following b.v.p.

$$\begin{cases} \dfrac{\partial}{\partial z}\left\{\phi(\mathbf{K})_{zz}\left[S_l \dfrac{k_{rl}(S_l)}{\mu_l}\left(\dfrac{\partial P_l}{\partial z} + \rho_l g\right)\right.\right. \\ \qquad \left.\left. + (1 - S_l)\dfrac{k_{rg}(1 - S_l)}{\mu_g}\left(\dfrac{\partial P_g}{\partial z} + \rho_g g\right)\right]\right\} = 0, \\ \\ P(0) = P_b, \\ \\ P(L) = P_{top}, \end{cases}$$

where P_b e P_{top} are known.

END OF THE FIRST PART

In the second part we will see some simplified problems aimed to a better understanding of time scales and reasonable modelling approaches.

Dinamic problem with phase separation

(Luca Meacci, 2009) Hypotheses

1. Geothermal fluid is pure H_2O (liquid or gas).

Dinamic problem with phase separation

(Luca Meacci, 2009) Hypotheses

1. Geothermal fluid is pure H_2O (liquid or gas).
2. Phases sharply separated by an interface $s(t)$ (no capillary effects, saturations can be only 0 or 1)

Dinamic problem with phase separation

(Luca Meacci, 2009) Hypotheses

1. Geothermal fluid is pure H_2O (liquid or gas).
2. Phases sharply separated by an interface $s(t)$ (no capillary effects, saturations can be only 0 or 1)
3. Rocks have constant porosity ϕ

Dinamic problem with phase separation

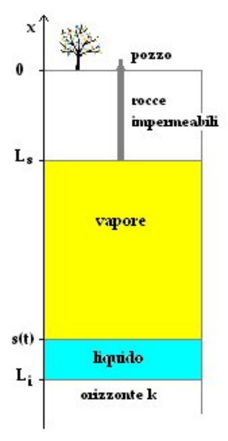
(Luca Meacci, 2009) Hypotheses

1. Geothermal fluid is pure H_2O (liquid or gas).
2. Phases sharply separated by an interface $s(t)$ (no capillary effects, saturations can be only 0 or 1)
3. Rocks have constant porosity ϕ
4. Permeability independent of temperature

Dinamic problem with phase separation

(Luca Meacci, 2009) Hypotheses

1. Geothermal fluid is pure H_2O (liquid or gas).
2. Phases sharply separated by an interface $s(t)$ (no capillary effects, saturations can be only 0 or 1)
3. Rocks have constant porosity ϕ
4. Permeability independent of temperature
5. 1-D geometry



The 1-D domain is $[L_i, L_s]$: $S_v = 1, S_w = 0$ above the free-boundary $s(t)$. $S_v = 0, S_w = 1$ below. Temperature changes linearly. At the upper boundary L_s pressure P_s is constant, at the lower boundary flux is null $(v_l = 0)$.

Parameter values

$L_s = -1300\, m$	$L_i = -3500\, m$
$T_s = 520\, {}^oK$	$T_i = 610\, {}^oK$
$s_{ip} \approx -3060\, m$	$T(s_{ip}) = 592\, {}^oK$
$\Delta L_v = L_s - s_{ip} \approx 1800\, m$	$\hat{T}(x \in [s_{ip}, L_s]) \approx 600^oK$
$\rho_{vc} = \frac{P^*(T(s_{ip}))}{rT} \approx 40Kg/m^3$	$\rho_l = 10^3 Kg/m^3$
$\phi = 10^{-2}$	$g = 9,8\, m/s$
$\mu_v \approx 2 \times 10^{-5} Pa \cdot s$	$K = 10^{-16} m^2$
$r = 4,6 \times 10^2 J/Kg^oK$	

Estimated Pressure:

$$P_v\,(x = s_{ip}) = P^*(T(x = s_{ip})) = P^*_{ip} \approx 1,1 \times 10^7\, Pa$$

$$P_v\,(x = L_s) = P_s = 3,1 \times 10^6\, Pa$$

$$\Delta P_v = P_v\,(x = L_s) - P_v\,(x = s_{ip}) \approx -8 \times 10^6\, Pa$$

Model eqs

for the liquid domain $L_i < x < s(t)$

Model eqs

for the liquid domain $L_i < x < s(t)$

- $\frac{\partial}{\partial t}(\phi\rho_l) + \frac{\partial}{\partial x}(\phi\rho_l v_l) = 0$, ($\rho_l$= constant). Consequently

Model eqs

for the liquid domain $L_i < x < s(t)$

- $\frac{\partial}{\partial t}(\phi\rho_l) + \frac{\partial}{\partial x}(\phi\rho_l v_l) = 0$, ($\rho_l$= constant). Consequently
- $\dfrac{\partial v_l}{\partial x} = 0 \qquad v_l|_{x=L_i} = 0$ which implies

Model eqs

for the liquid domain $L_i < x < s(t)$

- $\frac{\partial}{\partial t}(\phi\rho_l) + \frac{\partial}{\partial x}(\phi\rho_l v_l) = 0$, ($\rho_l$= constant). Consequently
- $\dfrac{\partial v_l}{\partial x} = 0 \qquad v_l|_{x=L_i} = 0$ which implies
- $v_l = 0$

Model eqs

for the liquid domain $L_i < x < s(t)$

- $\frac{\partial}{\partial t}(\phi\rho_l) + \frac{\partial}{\partial x}(\phi\rho_l v_l) = 0$, ($\rho_l =$ constant). Consequently
- $\dfrac{\partial v_l}{\partial x} = 0 \qquad v_l|_{x=L_i} = 0$ which implies
- $v_l = 0$
- Recalling Darcy's law $v_l = -\frac{K}{\phi\mu_l}\left(\frac{\partial P_l}{\partial x} + \rho_l g\right) = 0$,

Model eqs

for the liquid domain $L_i < x < s(t)$

- $\frac{\partial}{\partial t}(\phi\rho_l) + \frac{\partial}{\partial x}(\phi\rho_l v_l) = 0$, ($\rho_l$= constant). Consequently
- $\dfrac{\partial v_l}{\partial x} = 0 \qquad v_l|_{x=L_i} = 0$ which implies
- $v_l = 0$
- Recalling Darcy's law $v_l = -\frac{K}{\phi\mu_l}\left(\frac{\partial P_l}{\partial x} + \rho_l g\right) = 0$,
- pressure P_l solves $\dfrac{\partial P_l}{\partial x} = -\rho_l g, \qquad P_l(x = s(t)) = P_l(s(t))$, with $P_l(s(t))$ to be specified.

Model eqs

for the liquid domain $L_i < x < s(t)$

- $\frac{\partial}{\partial t}(\phi\rho_l) + \frac{\partial}{\partial x}(\phi\rho_l v_l) = 0$, ($\rho_l$= constant). Consequently
- $\dfrac{\partial v_l}{\partial x} = 0 \qquad v_l|_{x=L_i} = 0$ which implies
- $v_l = 0$
- Recalling Darcy's law $v_l = -\frac{K}{\phi\mu_l}\left(\frac{\partial P_l}{\partial x} + \rho_l g\right) = 0$,
- pressure P_l solves $\dfrac{\partial P_l}{\partial x} = -\rho_l g, \qquad P_l(x = s(t)) = P_l(s(t))$, with $P_l(s(t))$ to be specified.
- solution: $P_l(x) = \underbrace{P_l(s(t))}_{\text{ph. change press}} + \underbrace{\rho_l g(s(t) - x)}_{\text{hydrost. press}}$

Model eqs

for the vapor domain $s(t) < x < L_s$

Model eqs

for the vapor domain $s(t) < x < L_s$

- $\frac{\partial}{\partial t}\left(\phi\rho_v\right) + \frac{\partial}{\partial x}\left(\phi\rho_v v_v\right) = 0, \qquad s(t) < x < L_s$

Model eqs

for the vapor domain $s(t) < x < L_s$

- $\frac{\partial}{\partial t}(\phi\rho_v) + \frac{\partial}{\partial x}(\phi\rho_v v_v) = 0, \qquad s(t) < x < L_s$
- which, by means of ideal gas law for ρ_v, becomes

$$\frac{\partial}{\partial t}\left(\phi\frac{P_v}{rT}\right) + \frac{\partial}{\partial x}\left(\phi\frac{P_v}{rT}v_v\right) = 0 \tag{12}$$

Model eqs

for the vapor domain $s(t) < x < L_s$

- $\frac{\partial}{\partial t}\left(\phi\rho_v\right) + \frac{\partial}{\partial x}\left(\phi\rho_v v_v\right) = 0, \qquad s(t) < x < L_s$
- which, by means of ideal gas law for ρ_v, becomes

$$\frac{\partial}{\partial t}\left(\phi\frac{P_v}{rT}\right) + \frac{\partial}{\partial x}\left(\phi\frac{P_v}{rT}v_v\right) = 0 \qquad (12)$$

- but Darcy's law $v_v = -\frac{K}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x} + \rho_v g\right)$. still applies, so that

Model eqs

for the vapor domain $s(t) < x < L_s$

- $\frac{\partial}{\partial t}\left(\phi\rho_v\right) + \frac{\partial}{\partial x}\left(\phi\rho_v v_v\right) = 0, \qquad s(t) < x < L_s$
- which, by means of ideal gas law for ρ_v, becomes

$$\frac{\partial}{\partial t}\left(\phi\frac{P_v}{rT}\right) + \frac{\partial}{\partial x}\left(\phi\frac{P_v}{rT}v_v\right) = 0 \tag{12}$$

- but Darcy's law $v_v = -\frac{K}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x} + \rho_v g\right)$. still applies, so that
- $\frac{\partial}{\partial t}\left(\frac{P_v}{T}\right) - \frac{\partial}{\partial x}\left[\frac{P_v}{T}\frac{K}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x} + \frac{P_v}{rT}g\right)\right] = 0,$

Model eqs

for the vapor domain $s(t) < x < L_s$

- $\frac{\partial}{\partial t}\left(\phi\rho_v\right) + \frac{\partial}{\partial x}\left(\phi\rho_v v_v\right) = 0, \qquad s(t) < x < L_s$
- which, by means of ideal gas law for ρ_v, becomes

$$\frac{\partial}{\partial t}\left(\phi\frac{P_v}{rT}\right) + \frac{\partial}{\partial x}\left(\phi\frac{P_v}{rT}v_v\right) = 0 \qquad (12)$$

- but Darcy's law $v_v = -\frac{K}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x} + \rho_v g\right)$. still applies, so that
- $\frac{\partial}{\partial t}\left(\frac{P_v}{T}\right) - \frac{\partial}{\partial x}\left[\frac{P_v}{T}\frac{K}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x} + \frac{P_v}{rT}g\right)\right] = 0,$
- and assuming viscosity μ_v independent of temperature T

$$\frac{\partial P_v}{\partial t} - \frac{KT}{\phi\mu_v}\frac{\partial}{\partial x}\left[\frac{P_v}{T}\left(\frac{\partial P_v}{\partial x} + \frac{g}{r}\frac{P_v}{T}\right)\right] = 0.$$

Free boundary conditions

Free boundary conditions

Obviously we need to impose the **the continuity of mass flux** $[\rho_\beta (v_\beta - \dot{s})]_l^v = 0$.

Free boundary conditions

Obviously we need to impose the **the continuity of mass flux** $[\rho_\beta (v_\beta - \dot{s})]_l^v = 0$.

Since $v_l = 0$ it follows $\chi := \rho_v (v_v - \dot{s}) = -\rho_l \dot{s}$. Note that $\chi \cdot \dot{s} < 0$.

Meaning: χ is the **velocity of mass transfer through the interface** $s(t)$. Thus $\chi < 0$ means $\dot{s} > 0$, i.e. vapor condensation while $\chi > 0$ means that liquid boils.

Free boundary conditions

Obviously we need to impose the **the continuity of mass flux** $[\rho_\beta (v_\beta - \dot{s})]_l^v = 0$.

Since $v_l = 0$ it follows $\chi := \rho_v (v_v - \dot{s}) = -\rho_l \dot{s}$. Note that $\chi \cdot \dot{s} < 0$.

Meaning: χ is the **velocity of mass transfer through the interface** $s(t)$. Thus $\chi < 0$ means $\dot{s} > 0$, i.e. vapor condensation while $\chi > 0$ means that liquid boils.

Also we need to impose **the continuity of momentum flux**, i.e. $[\chi \vec{v}_\beta - \mathbf{T}_\beta \vec{n}]_l^v = 0$. where $\mathbf{T}_\beta$ is the stress tensor in phase β.

Free boundary conditions

Obviously we need to impose the **the continuity of mass flux** $[\rho_\beta\,(v_\beta - \dot{s})]_l^v = 0$.

Since $v_l = 0$ it follows $\chi := \rho_v\,(v_v - \dot{s}) = -\rho_l \dot{s}$. Note that $\chi \cdot \dot{s} < 0$.

Meaning: χ is the **velocity of mass transfer through the interface** $s(t)$. Thus $\chi < 0$ means $\dot{s} > 0$, i.e. vapor condensation while $\chi > 0$ means that liquid boils.

Also we need to impose **the continuity of momentum flux**, i.e. $[\chi \vec{v}_\beta - \mathbf{T}_\beta \vec{n}]_l^v = 0$. where $\mathbf{T}_\beta$ is the stress tensor in phase β.

Since $v_l = 0$ and $\mathbf{T}_\beta = P_\beta \mathbf{I}$, last conditions reduces to $\chi v_v = -\left(P_v - P_l\right)$.

Main question: which pressure at the phase change interface ?

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$,

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$, Then the continuity of momentum flux implies $\chi v_v = 0$, which holds iff

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$, Then the continuity of momentum flux implies $\chi v_v = 0$, which holds iff

Case 1 : $\chi = 0$.

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$, Then the continuity of momentum flux implies $\chi v_v = 0$, which holds iff

Case 1 : $\chi = 0$. We have $\rho_v (v_v - \dot{s}) = 0 \quad \Rightarrow \quad v_v|_{s(t)} = \dot{s}$ which implies

$$\rho_l \dot{s} = 0 \quad \Rightarrow \quad \dot{s} = 0$$

and so $v_v|_{s(t)} = \dot{s} = 0$. The free boundary does not move.

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$, Then the continuity of momentum flux implies $\chi v_v = 0$, which holds iff

Case 1 : $\chi = 0$. We have $\rho_v (v_v - \dot{s}) = 0 \quad \Rightarrow \quad v_v|_{s(t)} = \dot{s}$ which implies

$$\rho_l \dot{s} = 0 \quad \Rightarrow \quad \dot{s} = 0$$

and so $v_v|_{s(t)} = \dot{s} = 0$. The free boundary does not move.

Case 2 : $u_v|_{s(t)} = 0$.

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$, Then the continuity of momentum flux implies $\chi v_v = 0$, which holds iff

Case 1 : $\chi = 0$. We have $\rho_v (v_v - \dot{s}) = 0 \quad \Rightarrow \quad v_v|_{s(t)} = \dot{s}$ which implies

$$\rho_l \dot{s} = 0 \quad \Rightarrow \quad \dot{s} = 0$$

and so $v_v|_{s(t)} = \dot{s} = 0$. The free boundary does not move.

Case 2 : $u_v|_{s(t)} = 0$. Then the definition of χ implies $\rho_v \dot{s} = \rho_l \dot{s}$ which shows again that $v_v|_{s(t)} = \dot{s} = 0$.

Main question: which pressure at the phase change interface ?

Suppose $[P_\beta]_l^v = 0$, that is $P_v|_{s(t)} = P_l|_{s(t)} = P^*|_{s(t)}$, Then the continuity of momentum flux implies $\chi v_v = 0$, which holds iff

Case 1 : $\chi = 0$. We have $\rho_v (v_v - \dot{s}) = 0 \quad \Rightarrow \quad v_v|_{s(t)} = \dot{s}$ which implies

$$\rho_l \dot{s} = 0 \quad \Rightarrow \quad \dot{s} = 0$$

and so $v_v|_{s(t)} = \dot{s} = 0$. The free boundary does not move.

Case 2 : $u_v|_{s(t)} = 0$. Then the definition of χ implies $\rho_v \dot{s} = \rho_l \dot{s}$ which shows again that $v_v|_{s(t)} = \dot{s} = 0$.

In conclusion **moving** free boundary does not agree with a vanishing pressure jump at the interface. Thus we assume $[P_\beta]_l^v \neq 0$

A reasonable assumption: $\rho_v \ll \rho_l$.

[1]Recall that $\chi \cdot \dot{s} < 0$

A reasonable assumption: $\rho_v \ll \rho_l$. In this case $\chi \approx \rho_v v_v$ which in turn implies

$$\chi v_v\big|_{x=s(t)} = \rho_v v_v^2\big|_{s(t)} = P_l\big|_{s(t)} - P_v\big|_{s(t)} \,,$$

Some simple consequences:

[1] Recall that $\chi \cdot \dot{s} < 0$

A reasonable assumption: $\rho_v \ll \rho_l$. In this case $\chi \approx \rho_v v_v$ which in turn implies

$$\chi v_v\big|_{x=s(t)} = \rho_v v_v^2\big|_{s(t)} = P_l\big|_{s(t)} - P_v\big|_{s(t)},$$

Some simple consequences:

(1) $P_l\big|_{x=s(t)} \geq P_v\big|_{x=s(t)}$.

[1] Recall that $\chi \cdot \dot{s} < 0$

A reasonable assumption: $\rho_v \ll \rho_l$. In this case $\chi \approx \rho_v v_v$ which in turn implies

$$\chi v_v\big|_{x=s(t)} = \rho_v v_v^2\big|_{s(t)} = P_l\big|_{s(t)} - P_v\big|_{s(t)} ,$$

Some simple consequences:

(1) $P_l\big|_{x=s(t)} \geq P_v\big|_{x=s(t)}$.

(2) $\dot{s}$ and v_v have the opposite sign[1] ($s(t)$ moves downward if the vapor volumetric flux is upward and vice versa).

[1] Recall that $\chi \cdot \dot{s} < 0$

A reasonable assumption: $\rho_v \ll \rho_l$. In this case $\chi \approx \rho_v v_v$ which in turn implies

$$\chi v_v\big|_{x=s(t)} = \rho_v v_v^2\big|_{s(t)} = P_l\big|_{s(t)} - P_v\big|_{s(t)},$$

Some simple consequences:

(1) $P_l\big|_{x=s(t)} \geq P_v\big|_{x=s(t)}$.

(2) $\dot{s}$ and v_v have the opposite sign[1] ($s(t)$ moves downward if the vapor volumetric flux is upward and vice versa).

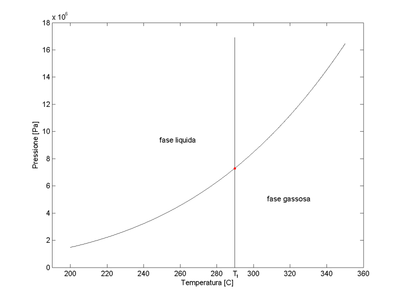
(3) $[P_\beta]_l^v$ can be estimated in terms of the mass flux.

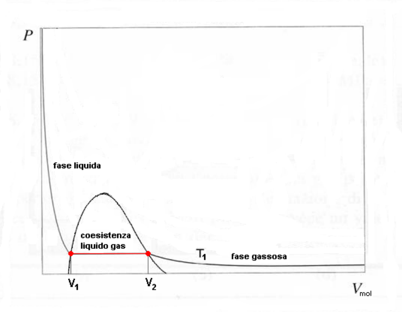
We **assume** $P_v\big|_{x=s(t)} = P^*\big|_{x=s(t)}$, P^* being the "saturated vapor pressure".

[1] Recall that $\chi \cdot \dot{s} < 0$

Saturated vapor pressure for H_2O

Clapeyron curve (left) and Andrews' diagram (right)





$$P^*(T) = 961,7\, exp\left\{17,35 \frac{T-273}{T}\right\},$$

(T in Kelvin, $961,7$ in Pa).

Liquid pressure

Previous assumption implies that

$$P_l|_{x=s(t)} = P_l(s(t)) = P^*|_{x=s(t)} + \rho_v v_v^2\big|_{x=s(t)} .$$

Liquid pressure

Previous assumption implies that

$$P_l|_{x=s(t)} = P_l(s(t)) = P^*|_{x=s(t)} + \rho_v v_v^2\big|_{x=s(t)} .$$

However

$$|v_v| \approx \frac{K}{\phi\mu_v}\left|\frac{\Delta P}{L}\right| \approx 10^{-6}\,\frac{m}{s}$$

Thus

$$\rho_v v_v^2 \approx 10^{-13}\,Pa$$

when ambient pressure is of order $10^6 \div 10^7\,Pa$!

Liquid pressure

Previous assumption implies that

$$P_l|_{x=s(t)} = P_l(s(t)) = P^*|_{x=s(t)} + \rho_v v_v^2\Big|_{x=s(t)} .$$

However

$$|v_v| \approx \frac{K}{\phi \mu_v} \left| \frac{\Delta P}{L} \right| \approx 10^{-6}\, \frac{m}{s}$$

Thus

$$\rho_v v_v^2 \approx 10^{-13}\, Pa$$

when ambient pressure is of order $10^6 \div 10^7\, Pa$!

We conclude that the pressure jump at the interface is very small and consequently $\dot{s} \approx 0$ (but we have seen before that if it is totally neglected then $s(t)$ remains steady).

We can still consider

$$P_l|_{x=s(t)} = P_v|_{x=s(t)} = P^*(T(x))|_{x=s(t)}\,.$$

i.e. pressure jump equal to zero, **but we need to consider two different time scales** to account for the movement of the interface!

Conclusion:

We can still consider

$$P_l|_{x=s(t)} = P_v|_{x=s(t)} = P^*(T(x))|_{x=s(t)} \, .$$

i.e. pressure jump equal to zero, **but we need to consider two different time scales** to account for the movement of the interface!

Conclusion: **The dynamics of vapor diffusion in $[s(t), L_s]$ occurs over a time scale for which the free boundary appears at rest.**

We can still consider

$$P_l|_{x=s(t)} = P_v|_{x=s(t)} = P^*(T(x))|_{x=s(t)}\,.$$

i.e. pressure jump equal to zero, **but we need to consider two different time scales** to account for the movement of the interface!

Conclusion: **The dynamics of vapor diffusion in $[s(t), L_s]$ occurs over a time scale for which the free boundary appears at rest.**

Vice versa over the time scale where the movement of the interface is not negligible diffusive effects appears at equilibrium.

We can still consider

$$P_l|_{x=s(t)} = P_v|_{x=s(t)} = P^*(T(x))|_{x=s(t)} .$$

i.e. pressure jump equal to zero, **but we need to consider two different time scales** to account for the movement of the interface!

Conclusion: **The dynamics of vapor diffusion in $[s(t), L_s]$ occurs over a time scale for which the free boundary appears at rest.**

Vice versa over the time scale where the movement of the interface is not negligible diffusive effects appears at equilibrium.

This suggests to use a *quasi stationary* approach to study the movement of $s(t)$.

Free boundary equation

Apply Darcy's law to the equation of mass flux continuity through the interface. to get

$$\dot{s}\left(1-\frac{\rho_v}{\rho_l}\right)=\frac{\rho_v}{\rho_l}\frac{K_v}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x}+\rho_v g\right)\Bigg|_{x=s(t)}. \qquad (13)$$

Then recalling the assumption about the vapor pressure at interface the free boundary eq. writes

$$\dot{s}\left(1-\frac{P^*(s(t))}{rT\rho_l}\right)=\frac{P^*(s(t))}{rT\rho_l}\frac{K_v}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x}+\frac{P^*(s(t))}{rT}g\right)\Bigg|_{x=s(t)}. \qquad (14)$$

Free boundary prob. with $P_v\big|_{x=s(t)} = P^*\big|_{x=s(t)}$,

$$
\begin{cases}
\dfrac{\partial P_v}{\partial t} - \dfrac{K\,T}{\phi\mu_v}\dfrac{\partial}{\partial x}\left[\dfrac{P_v}{T}\left(\dfrac{\partial P_v}{\partial x} + \dfrac{g}{r}\dfrac{P_v}{T}\right)\right] = 0, \\[2ex]
P_v(x = L_s) = P_s, \\[2ex]
P_v(x = s(t)) = P^*(s(t)), \\[2ex]
\dot{s}\left(1 - \dfrac{P^*(s(t))}{rT\rho_l}\right) = \dfrac{P^*(s(t))}{rT\rho_l}\dfrac{K}{\phi\mu_v}\left(\dfrac{\partial P_v}{\partial x} + \dfrac{P^*(s(t))}{rT}g\right)\Bigg|_{x=s(t)}, \\[2ex]
P_v(t = 0) = P_{in}(x), \\[2ex]
s(t = 0) = s_{in},
\end{cases}
$$

Here $T(x)$ is known, $P^*(x) = P^*(T(x))$, $P_{in}(x)$ and s_{in} are initial values.

Possible simplifications for typical values

$$\beta_1(x) := \frac{\frac{P_v}{rT} g}{\frac{\partial P_v}{\partial x}} = \frac{\text{gravitational force}}{\text{pressure gradient}} \approx \frac{\frac{P_v}{rT} g}{\frac{\Delta P_v}{\Delta L_v}} \approx 10^{-1}$$

Possible simplifications for typical values

$$\beta_1(x) := \frac{\frac{P_v}{rT}g}{\frac{\partial P_v}{\partial x}} = \frac{\text{gravitational force}}{\text{pressure gradient}} \approx \frac{\frac{P_v}{rT}g}{\frac{\Delta P_v}{\Delta L_v}} \approx 10^{-1}$$

We cannot neglet the gravitational contribution!

Possible simplifications for typical values

$$\beta_1(x) := \frac{\frac{P_v}{rT} g}{\frac{\partial P_v}{\partial x}} = \frac{\text{gravitational force}}{\text{pressure gradient}} \approx \frac{\frac{P_v}{rT} g}{\frac{\Delta P_v}{\Delta L_v}} \approx 10^{-1}$$

We cannot neglet the gravitational contribution! Consider then the ratio *vapor density/liquid density* at the interface:

$$\beta_2 := \frac{\rho_v(x = s(t))}{\rho_l} = \frac{P^*(s(t))}{rT(s(t))\rho_l} \approx \frac{P^*_{ip}}{r\hat{T}\rho_l} \approx 10^{-2}$$

This justifies the assumption $\rho_v \ll \rho_l$ made before.

Possible simplifications for typical values

$$\beta_1(x) := \frac{\frac{P_v}{rT}g}{\frac{\partial P_v}{\partial x}} = \frac{\text{gravitational force}}{\text{pressure gradient}} \approx \frac{\frac{P_v}{rT}g}{\frac{\Delta P_v}{\Delta L_v}} \approx 10^{-1}$$

We cannot neglet the gravitational contribution! Consider then the ratio *vapor density/liquid density* at the interface:

$$\beta_2 := \frac{\rho_v(x = s(t))}{\rho_l} = \frac{P^*(s(t))}{rT(s(t))\rho_l} \approx \frac{P^*_{ip}}{r\hat{T}\rho_l} \approx 10^{-2}$$

This justifies the assumption $\rho_v \ll \rho_l$ made before. We the neglect β_2 to get a simplified free boundary equation

$$\dot{s} = \frac{P^*(s(t))}{rT\rho_l}\frac{K}{\phi\mu_v}\left(\frac{\partial P_v}{\partial x} + \frac{P^*(s(t))}{rT}g\right)\Bigg|_{x=s(t)}.$$

Equations scaling

Take $t_{\mathrm{diff}} := \frac{\phi\mu_v L^2}{KP_{ip}^*}$ as a characteristic diffusion time and $t_s := \frac{\rho_l}{\rho_{vc}} t_{diff}$ as a characteristic interface time. For typical values $t_{\mathrm{diff}} \approx 8,4 \times 10^8\, s \approx 27$ years, and $\rho_l/\rho_{vc} \approx 25$. Thus t_s is of order hundreds of years.

Equations scaling

Take $t_{\text{diff}} := \frac{\phi \mu_v L^2}{K P^*_{ip}}$ as a characteristic diffusion time and $t_s := \frac{\rho_l}{\rho_{vc}} t_{diff}$ as a characteristic interface time. For typical values $t_{\text{diff}} \approx 8,4 \times 10^8 \, s \approx 27$ years, and $\rho_l / \rho_{vc} \approx 25$. Thus t_s is of order hundreds of years. If we take as scale time *that of the interface*, then the scaled (adimensionalized) equations appear as follows

Equations scaling

Take $t_{\text{diff}} := \frac{\phi \mu_v L^2}{K P_{ip}^*}$ as a characteristic diffusion time and $t_s := \frac{\rho_l}{\rho_{vc}} t_{diff}$ as a characteristic interface time. For typical values $t_{\text{diff}} \approx 8,4 \times 10^8\, s \approx 27$ years, and $\rho_l/\rho_{vc} \approx 25$. Thus t_s is of order hundreds of years. If we take as scale time *that of the interface*, then the scaled (adimensionalized) equations appear as follows

$$\underbrace{\frac{t_{\text{diff}}}{t_s}}_{\frac{\rho_{vc}}{\rho_l} \approx 4 \times 10^{-2}} \frac{\partial \tilde{P}_v}{\partial \tilde{t}} - \tilde{T} \frac{\partial}{\partial \tilde{x}} \left[\frac{\tilde{P}_v}{\tilde{T}} \left(\frac{\partial \tilde{P}_v}{\partial \tilde{x}} + \alpha \frac{\tilde{P}_v}{\tilde{T}} \right) \right] = 0,$$

Equations scaling

Take $t_{\text{diff}} := \frac{\phi \mu_v L^2}{K P^*_{ip}}$ as a characteristic diffusion time and $t_s := \frac{\rho_l}{\rho_{vc}} t_{diff}$ as a characteristic interface time. For typical values $t_{\text{diff}} \approx 8,4 \times 10^8\, s \approx 27$ years, and $\rho_l/\rho_{vc} \approx 25$. Thus t_s is of order hundreds of years. If we take as scale time *that of the interface*, then the scaled (adimensionalized) equations appear as follows

$$\underbrace{\frac{t_{\text{diff}}}{t_s}}_{\frac{\rho_{vc}}{\rho_l} \approx 4 \times 10^{-2}} \frac{\partial \tilde{P}_v}{\partial \tilde{t}} - \tilde{T} \frac{\partial}{\partial \tilde{x}} \left[\frac{\tilde{P}_v}{\tilde{T}} \left(\frac{\partial \tilde{P}_v}{\partial \tilde{x}} + \alpha \frac{\tilde{P}_v}{\tilde{T}} \right) \right] = 0,$$

$$\dot{\tilde{s}} = \frac{\tilde{P}^*(\tilde{x})}{\tilde{T}} \left(\frac{\partial \tilde{P}_v}{\partial \tilde{x}} + \alpha \frac{P^*(\tilde{x})}{T} \right) \Bigg|_{\tilde{x} = \tilde{s}(\tilde{t})} \qquad \left(\alpha := \frac{gL}{rT_i} \approx 10^{-1} \right)$$

Quasi-steady problem

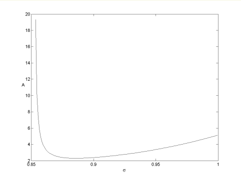
$$
\begin{cases}
\dfrac{\partial}{\partial \tilde{x}}\left[\dfrac{\tilde{P}_v}{\tilde{T}}\left(\dfrac{\partial \tilde{P}_v}{\partial \tilde{x}}+\alpha\dfrac{\tilde{P}_v}{\tilde{T}}\right)\right]=0,\\[2ex]
\tilde{P}_v(\tilde{x}=0)=\tilde{P}_s,\\[2ex]
\tilde{P}_v(\tilde{x}=\tilde{s}(\tilde{t}))=\tilde{P}^*(\tilde{s}(\tilde{t})),\\[2ex]
\dot{\tilde{s}}=\dfrac{\tilde{P}^*(\tilde{x})}{\tilde{T}}\left.\left(\dfrac{\partial \tilde{P}_v}{\partial \tilde{x}}+\alpha\dfrac{\tilde{P}^*(\tilde{x})}{T}\right)\right|_{\tilde{x}=\tilde{s}(\tilde{t})},\\[2ex]
\tilde{P}_v(\tilde{t}=0)=\tilde{P}_{in}(\tilde{x}),\\[2ex]
\tilde{s}(\tilde{t}=0)=\tilde{s}_0.
\end{cases}
\tag{15}
$$

Some implications

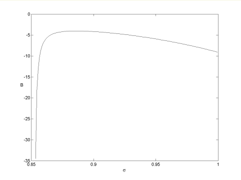
Being T a known linear function of x the quasi-steady problem can be written as a function of T and easily integrated:

$$P = \sqrt{\frac{A}{1-\delta}T^2 + BT^{2\delta}}, \qquad \delta = \alpha/(1-T_s)$$

$T_s = 0.85$ is the temperature at basin top boundary, A and B are known functions of $\sigma(t) := T(s(t))$.



(g) A as a function of σ



(h) B as a function of σ

Both functions diverges as $\sigma \to T_s$.

Reparametrized free boundary eq

$$\dot{\sigma} = A(\sigma)\gamma^2, \qquad \sigma(0) = \sigma_0,$$

If $s(t)$ is sufficiently far from the the basin top boundary then $A(\sigma) \approx A_0 - A(s_0)$ and

$$\sigma(t) = A_0\gamma^2 t + \sigma_0,$$

This allows the characteristic time of the moving boundary can be better estimated

$$t_{s,new} := \frac{t_s}{A_0\gamma^2} \approx \frac{\rho_l}{\rho_{vc}} \frac{1}{A_0\gamma^2}\, t_{diff}$$

Being $A_0 \approx 4$ and $\gamma^2 \approx 2,2 \times 10^{-2}$ we get

$$t_{s,new} \approx 2,8 \times 10^2\, t_{diff} \approx 7500\ years$$

which appears compatible with geological estimates: this seems to be the time needed for the Larderello basin to evolve from water dominated to vapor dominated.

Numerical simulations (unsteady problem)

Use now the diffusive time scale.

$$
\begin{cases}
\dfrac{\partial P}{\partial t} - \dfrac{1}{T}\dfrac{\partial}{\partial x}\left[\dfrac{P_v}{T}\left(\dfrac{\partial P_v}{\partial x} + \alpha \dfrac{P_v}{T}\right)\right] = 0, \\
P_v(x=0) = P_s, \\
P_v(x=s(t)) = P^*(s(t)), \\
\dot{s} = \beta \dfrac{P^*(x)}{T}\left(\dfrac{\partial P_v}{\partial x} + \alpha \dfrac{P^*(x)}{T}\right)\Bigg|_{x=s(t)}, \\
P_v(t=0) = P_0(x), \\
s(t=0) = s_0.
\end{cases}
$$

where

$$\beta = \frac{P^*_{ip}}{rT_i\rho_l}$$

and $P_0(x)$, s_0 given i. c.

Dimensionless input data

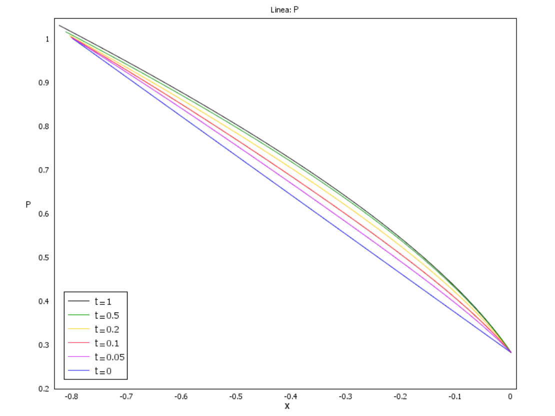
$L_s = 0$	$L_i = -1$
$T_s = 0,8525$	$T_i = 1$
$s_0 = -0,8$	$T(s_0) = 0,9705$
$P_s = 0,2818$	$P^*_{ip}(t=0) = 1$
$\alpha = 0.0768$	$\beta = 0.0392$

Table: Larderello simulation with diffusive scaling time

Pressure initial condition

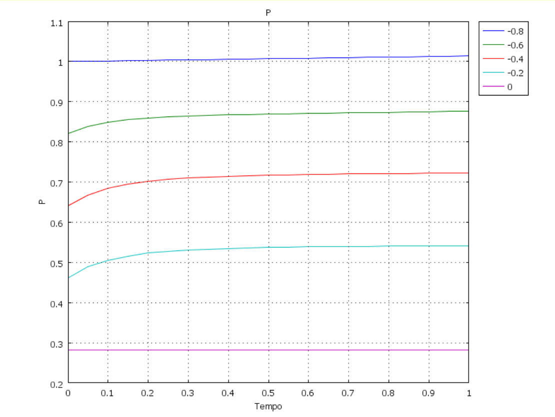
$$P_0(x) = P_s - (1 - P_s)(-0,2 + x).$$

Vapour pressure w.r.t. depth



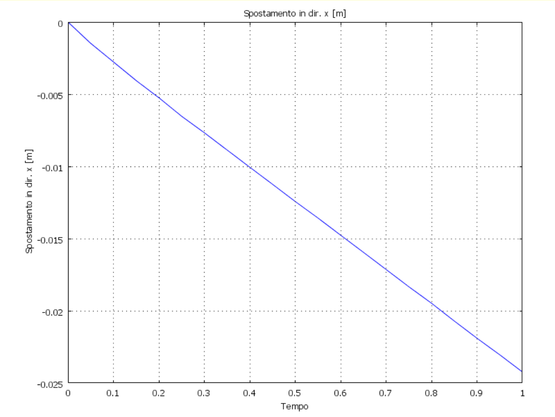
Pressure w.r.t. depth x at different times. The domain of definition increases with time (the free boundary moves downwards).

Vapour pressure w.r.t. time



Pressure w.r.t. time t at different depths. .

Moving boundary



Interface boundary vs. time: $s(t)$ moves downwards.

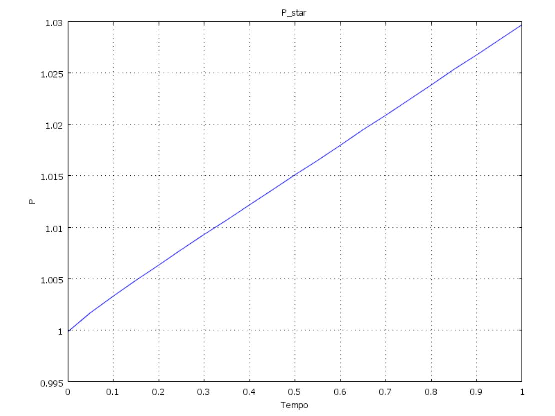
Some conclusions

At time $t = 2250$ years the moving boundary reaches the bottom of the geothermal basin.

Although simplified, this analysis shows that diffusive effects have a characteristic time of decades while the free boundary needs thousands of years to vanish

This appears compatible with geological studies: Larderello is an “old”basin. Thousands of years ago it was a water dominated basin which has now turned to a vapor dominated one.

Clapeyron pressure at the interface



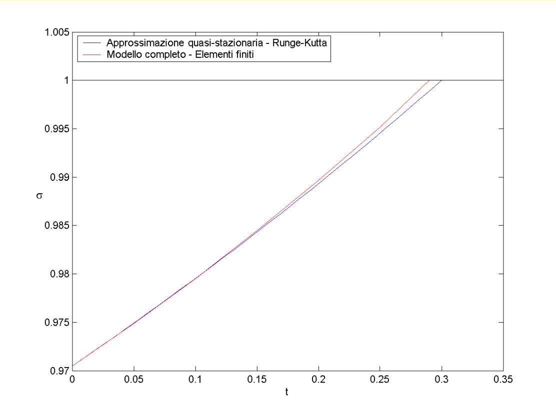
The interface pressure increases with time since temperature increases with depth.

Larderello simulation with moving boundary scaling time

$$\begin{cases} \beta \dfrac{\partial P}{\partial t} - \dfrac{1}{T}\dfrac{\partial}{\partial x}\left[\dfrac{P_v}{T}\left(\dfrac{\partial P_v}{\partial x} + \alpha \dfrac{P_v}{T}\right)\right] = 0, \\ P_v(x=0) = P_s, \\ P_v(x = s(t)) = P^*(s(t)), \\ \dot{s} = \dfrac{P^*(x)}{T}\left(\dfrac{\partial P_v}{\partial x} + \alpha \dfrac{P^*(x)}{T}\right)\Bigg|_{x=s(t)}, \\ P_v(t=0) = P_0(x), \\ s(t=0) = s_0. \end{cases}$$

Input data are the same as before

Comparison with the quasi-steady problem



A true well

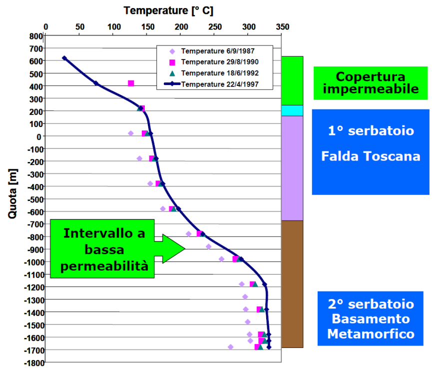


Figure: A typical temperature profile (Bagnore, Amiata Volcano)

Convective motions

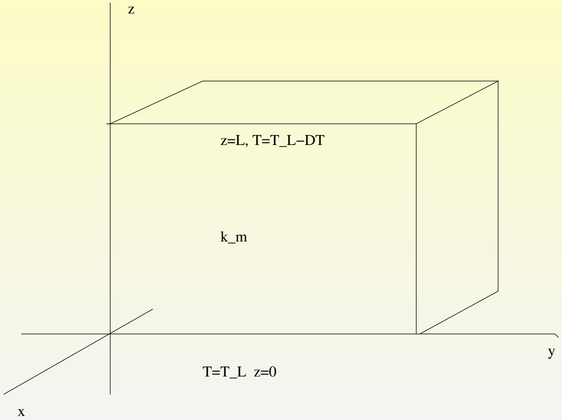
(Matteo Cerminara, 2009) The temperature well profile suggests the possibility of convective (efficient) motions where ∇T is very small.

Navier-Stokes eqn. in porous media (Oberbeck-Boussinesq approx.)

$$\begin{cases} \nabla \cdot \mathbf{q}(\mathbf{x},t) = 0 \\ \\ \varrho_w \left(\partial_t + \mathbf{q}(\mathbf{x},t)\cdot\nabla\right) q(\mathbf{x},t) = -\nabla p(\mathbf{x},t) \\ \quad + \varrho_w g \beta \left(T(\mathbf{x},t) - T_L + \Delta T\right) \nabla z - \dfrac{\mu_w}{k_m}\mathbf{q}(\mathbf{x},t) \\ \\ \langle \varrho c \rangle \partial_t T(\mathbf{x},t) = \langle \lambda \rangle \nabla^2 T(\mathbf{x},t) - \varrho_w c_w \mathbf{q}(\mathbf{x},t)\cdot\nabla T(\mathbf{x},t) \end{cases}$$

where $\langle \varrho c \rangle := \phi \varrho_w c_w + (1-\phi)\varrho_m c_m$, $\langle \lambda \rangle := \phi \lambda_w + (1-\phi)\lambda_m$, and $p(\mathbf{x},t) := P(\mathbf{x},t) - P_{\mathrm{hy}}(z)$

Geometry



Question: are convective cells possible?

There exists a steady conductive: (dimensionless variables)

$$\begin{cases} \mathbf{q} & = 0 \\ T & = 1 - z \\ p & = -C(1-z)^2 \end{cases}$$

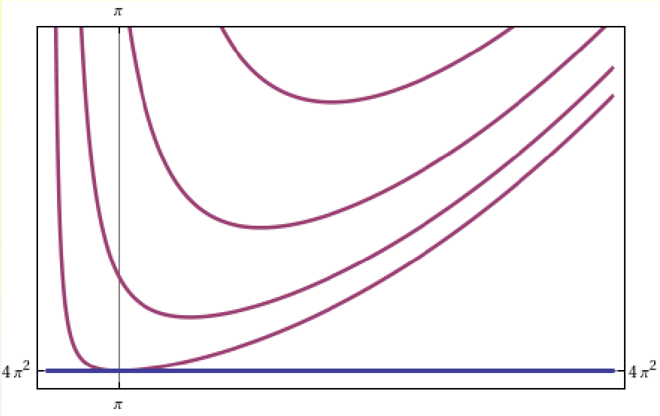
where C is a given parameter.

Linear stability analysis shows that the above solution may be unstable. Fourier modes may develop if

$$Ra := \frac{c_w \varrho_w^2 g \beta \Delta T L k_m}{\mu_m \langle \lambda \rangle} > \frac{(\boldsymbol{\xi}^2 + j^2\pi^2)^2}{\boldsymbol{\xi}^2}$$

where $\boldsymbol{\xi}$ is the horizontal wave-number and j is any integer.

Linear stability



The r.h.s. minimum is $4\pi^2$. Many physical parameters have a well defined value so that the instability conditions rewrites simply as

$$k_m \Delta T L > 2.6 \times 10^{10} m^{3\circ} C$$

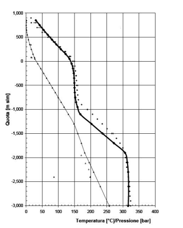
and being $\Delta t \simeq 100^\circ C$ and $L \simeq 1000m$ we obtain the lower limit for permeability, above which convective motions may occur:

$$k_m > 1\, md$$

Above this value water has the necessary mobility to generate convective cells. This appear in agreement with geological estimates.

The experimental temperature profile (Bagnore well)

Measured temperature profiles seem to validate the existence of convective motions.



There are two clearly distinct zones: one where the geothermal gradient $\eta \approx 0.15°Cm^{-1}$ (conductive), the other where $\eta \approx 0.02°Cm^{-1}$ (convective): there is one order of magnitude difference between the two values! In this conditions the linear approach (small perturbations) cannot be applied.

Thus the Rayleigh approach does not justify completely the heat flux measured! Indeed it can be proved that the mean vertical heat flux is (at first order) the same as the conductive one.

Consider then the Navier-Stokes eqn. in porous media (Oberbeck-Boussinesq approx.) in the steady case with the inertial term neglected:

$$\begin{cases} \nabla \cdot \mathbf{q}(\mathbf{x},t) = 0 \\ \\ 0 = -\nabla p(\mathbf{x},t) + \varrho_w g \beta \left(T(\mathbf{x},t) - T_L + \Delta T \right) \nabla z - \dfrac{\mu_w}{k_m} \mathbf{q}(\mathbf{x},t) \\ \\ 0 = \langle \lambda \rangle \nabla^2 T(\mathbf{x},t) - \varrho_w c_w q(\mathbf{x},t) \cdot \nabla T(\mathbf{x},t) \end{cases}$$

Boundary conditions

Take

$$q_z(x,y,0) = q_z(x,y,L) = 0, \quad T(x,y,0) = T_L, \quad T(x,y,L) = T_L - \Delta T.$$

It can be proved that the mean vertical heat flux $\langle J_z \rangle_S$ (which is a function of z only) is preserved, being the "mean" defined as

$$\langle f(x,y,z) \rangle_S = \frac{1}{S} \int_S f(x,y,z)\, dx dy$$

where S is any sufficiently extended horizontal surface. Thus

$$-\underbrace{\langle \lambda \rangle \partial_z \langle T(x,y,z) \rangle_S}_{\text{conductive}} + \underbrace{\varrho_w c_w \langle T q_z \rangle_S}_{\text{convective}} = \text{constant}$$

Idea

The transition from the convective to conductive zone is not sharp! The temperature gradient changes smoothly accordingly with the convective heat flux, maintaining constant the total flux!

Idea

The transition from the convective to conductive zone is not sharp! The temperature gradient changes smoothly accordingly with the convective heat flux, maintaining constant the total flux! One analytic solution that fits this idea is:

$$q_x = q_y = 0, \quad q_z = BT_2 \cos(\boldsymbol{\xi} \cdot \mathbf{r}), \qquad \mathbf{r} := (\mathbf{x}, \mathbf{y})$$

$$T = T_1 - \eta z + T_2 \cos(\boldsymbol{\xi} \cdot \mathbf{r}),$$

$$p = p_1 + (B/A)(T_1 z \tfrac{1}{2}\eta z^2), \qquad \boldsymbol{\xi}^2 = (B/D)\eta$$

however this solution does not fit the boundary conditions but, unlike the Rayleigh solution, the vertical mean convective flux is not zero: in dimensional form

$$\langle J_z \rangle_S = \langle \lambda \rangle \left(\eta + \frac{k_m T_2^2}{2C_1} \right)$$

The extra term is just the heat transported by convective motions. Notice that as $T_2 \to 0$ we get again the conductive solution and pressure remains the same independently of absence or presence of convective motions.

Fictitious boundary conditions:

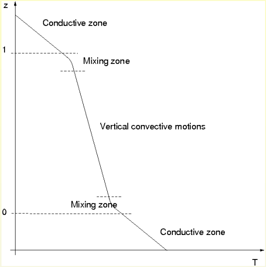
$$q_z(x, y, 0) = q_z(x, y, L) = \frac{\varrho_w g \beta k_m}{\mu_w} T_2 \cos(\boldsymbol{\xi} \cdot \mathbf{r}),$$

$$T(x, y, 0) = T_1 + T_2 \cos(\boldsymbol{\xi} \cdot \mathbf{r}),$$

$$T(x, y, L) = T_L - \eta L + T_2 \cos(\boldsymbol{\xi} \cdot \mathbf{r}).$$

These are the b.c. the would attained by the previous analytic solution. Since they differ from the original ones, we are forced to allow that solution only in a narrowed layer $(z_0, z_1) \subset (0, 1)$.

Mixing zones



It can be proved that the analytic solution is unique in the narrower convective region. Due to the constancy of $\langle J_z\rangle_S = -D\partial_z\langle T\rangle_S + \langle Tq_z\rangle_s$ =constant, in the *mixing zones* q_z decreases and $\partial_z\langle T\rangle_S$ increases so that the boundary conditions have time to adjust to the right ones.

This procedure has been tested using field data. In particular

This procedure has been tested using field data. In particular

- order of magnitude of all relevant parameters fit the measured values (for example specific discharge w is of order 10^{-8} m/s)

This procedure has been tested using field data. In particular

- order of magnitude of all relevant parameters fit the measured values (for example specific discharge w is of order 10^{-8} m/s)
- convective motions seem to have very little influence on the hydrostatic pressure

This procedure has been tested using field data. In particular

- order of magnitude of all relevant parameters fit the measured values (for example specific discharge w is of order 10^{-8} m/s)
- convective motions seem to have very little influence on the hydrostatic pressure
- the dimension of convective cells depends on the Rayleigh number: as Ra increases new modes may grow and the cell dimension L_{cell} decreases more and more.

This procedure has been tested using field data. In particular

- order of magnitude of all relevant parameters fit the measured values (for example specific discharge w is of order 10^{-8} m/s)
- convective motions seem to have very little influence on the hydrostatic pressure
- the dimension of convective cells depends on the Rayleigh number: as Ra increases new modes may grow and the cell dimension L_{cell} decreases more and more.
- the characteristic time $t_{char} = L_{\text{cell}}/w$ of convective motions is estimated to be ≈ 1000 years.

Work in progress

So far things appear easy but I didn't mention

- The phase equilibrium stability approach

Each of these subjects (actually under investigation) would require a dedicated lecture and even the recent literature is controversial.

This justifies the full re-examination of the whole problem that is at the base of the MACGEO project.

Work in progress

So far things appear easy but I didn't mention

- The phase equilibrium stability approach
- The choice of the *right* EOS

Each of these subjects (actually under investigation) would require a dedicated lecture and even the recent literature is controversial.

This justifies the full re-examination of the whole problem that is at the base of the MACGEO project.

Work in progress

So far things appear easy but I didn't mention

- The phase equilibrium stability approach
- The choice of the *right* EOS
- The IMPES approach

Each of these subjects (actually under investigation) would require a dedicated lecture and even the recent literature is controversial.

This justifies the full re-examination of the whole problem that is at the base of the MACGEO project.

Work in progress

So far things appear easy but I didn't mention

- The phase equilibrium stability approach
- The choice of the *right* EOS
- The IMPES approach
- The role of capillary pressure

Each of these subjects (actually under investigation) would require a dedicated lecture and even the recent literature is controversial.

This justifies the full re-examination of the whole problem that is at the base of the MACGEO project.

Work in progress

So far things appear easy but I didn't mention

- The phase equilibrium stability approach
- The choice of the *right* EOS
- The IMPES approach
- The role of capillary pressure
- The numerical procedure

Each of these subjects (actually under investigation) would require a dedicated lecture and even the recent literature is controversial.

This justifies the full re-examination of the whole problem that is at the base of the MACGEO project.

C. R. Faust and J. W. Mercer.
Geothermal Reservoir Simulation 1. Mathematical Models for Liquid- and Vapor-Dominated Hydrothermal Systems.
Water Resources Research, 15:23–30, 1979.

G. G. Tsypkin, C. Calore, and M. Marcolini.
Mathematical modeling of cold water injection into a depleted high-temperature geothermal reservoir.
HIGH TEMPERATURE, 44(3):450–457, MAY-JUN 2006.

AA Barmin and GG Tsypkin.
Moving phase transition front at water injection into vapor saturated reservoir.
DOKLADY AKADEMII NAUK, 350(2):195–197, SEP 1996.

Andrew W. Woods and Shaun D. Fitzgerald.
The vaporization of a liquid front moving through a hot porous rock.
Journal of Fluid Mechanics Digital Archive, 251(-1):563–579, 2006.

S. D. Fitzgerald and A. W. Woods.
Instabilities during liquid migration into superheated geothermal reservoirs.
Water Resources Research, 34:2089–2101, 1998.

MJ O'Sullivan.
Geothermal reservoir simulation.
Energy Research, 9:313–332, 1985.

SP White.
Multiphase nonisothermal transport of systems of reacting chemicals.
Water Resources Research, 31(7):1761–1772, 1995.

KH Coats.
Geothermal Reservoir Modeling, paper SPE-6892, presented at the 52nd Annual Fall Technical Conference and Exhibition of the SPE, Denver, Colorado, 1977.

D. H. Brownell, Jr., S. K. Garg, and J. W. Pritchett.
Governing Equations for Geothermal Reservoirs.
Water Resources Research, 13:929–934, 1977.