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ABSTRACTS

Gennadiy Averkov

University of Magdeburg

On new partial answers to Matheron's conjecture and related results

The covariogram of a convex body K in the Euclidean space E^n is the function which associates to each vector $u \in E^n$ the volume of $K \cap (K + u)$. In 1986 Matheron asked whether for $n = 2$ the covariogram data of K determine K uniquely, up to translations and reflections. Matheron's conjecture is relevant in a big number of applied disciplines, such as stereology, image analysis, and crystallography. We sharpen some of the known results on Matheron's conjecture, indicating how much of the covariogram information is needed to get the uniqueness of determination. Further on, we extend the class of convex bodies for which the covariogram conjecture is confirmed by including all planar convex bodies possessing two non-degenerate boundary arcs being reflections of each other.

Franck Barthe

Université Paul-Sabatier, Toulouse

Lipschitz functions which vary the most

Motivated by moderate and large deviations principles for the two sample matching problem (a random version of optimal transport) we consider geometric optimization problems where one looks for 1-Lipschits functions on a domain with maximal global variation. In the case of the ball, an answer is made possible by spectral analysis. Many questions remain for convex sets.

Jesús Bastero

Universidad de Zaragoza

High dimensional sections of symmetric convex bodies

Let K be a centrally symmetric convex body of volume one in isotropic position. Hensley in the seventies proved that

$$\frac{1}{\sqrt{12}} \le L_K \; |H_\theta \cap K|_{n-1} \le \frac{1}{\sqrt{2}}$$

where H_θ is the hyperplane orthogonal to $\theta \in S^{n-1}$. In this talk I will survey related results for different dimensions, including the very recent improvement of the inequality above in a joint work with D. Alonso, J. Bernués and G. Paouris:

$$\frac{1-\varepsilon}{\sqrt{2\pi}} \le L_K \; |E \cap K|_{n-k}^{1/k} \le \frac{1+\varepsilon}{\sqrt{2\pi}}$$

that holds for random $(n-k)$-dimensional subspaces E of $\mathbf{R}^n$, whenever $1 \le k \le C_1 \varepsilon \log n / (\log \log n)^2$, ($C_1 > 0$ is an absolute constant).

Chiara Bianchini

Università degli studi di Firenze

Minkowski addition of functions and quasi-concavity of solution to elliptic equations

(joint work with P. Salani)

We investigate on conditions which guarantee that, in a Dirichlet problem of elliptic type, relevant geometric properties of the domain are inherited by the level sets of its solutions. In particular we are interested in finding assumptions on the operator which assure that the solution has convex level sets. We deal with the following problem:

$$\begin{cases} F(x,u,Du,D^2u)=0 & \text{in } \Omega \\ u=0 & \text{on } \partial\Omega_0 \\ u=1 & \text{on } \partial\Omega_1 \,, \end{cases} \tag{1}$$

We generalize some previous results proving that, under quite general assumption on F the solution of (1) has convex super-level sets.

The main tool of our proof is the so called Minkowski addition of functions. Roughly speaking the addition $u = u_1 + u_2$ means that the super-level sets of u are the Minkowski sum of the corresponding super-level sets of u_1 and u_2.

Károly J. Böröczky

Alfréd Rényi Institute of Mathematics, Budapest

Stability of geometric inequalities

Let K be a convex body in $\mathbb{R}^n$; namely, a compact convex set with non–empty interior. Since the days of H. Minkowski, geometric inequalities for convex bodies are of high interest, and the stability versions of many have been established. The list of contributors include H. Minkowski, H. Groemer, V.I. Diskant, Peter M. Gruber and Rolf Schneider, and many others. In this talk, I survey the classical results, present some recent achievements, and discuss applications. Finally I pose some problems.

Paolo Dulio

Politecnico di Milano

Symmetries arising from X-rays

In a given space Y, the uniqueness problem of Geometric Tomography asks for the minimum number of radiographies needed for the unique determination of a geometric object in Y. Usually the problem is addressed to the class $\mathcal{C}_Y$ of all convex bodies of Y, but also other classes can be considered. A crucial step is to highlight geometric structures linked to possible ambiguities in the reconstruction process.

When radiographies are performed in $\mathcal{C}_Y$ through parallel X-rays in a set U of directions, the so called U-polygons come out, and, if we look beyond convexity, these generalize to switching components. When radiographies are taken through point X-rays at a set P of sources, P-polygons appear.

Understanding the geometric structure of such sets often allow uniqueness results to be achieved. For instance, in the Euclidean plane $\mathbb{R}^2$, no U-polygons exist if U consists of 4 directions with transcendental cross-ratio ρ. This means that such a set of four directions uniquely determines a convex body in $\mathcal{C}_{\mathbb{R}^2}$ ([4]). The same is true for the class $\mathcal{C}_{\mathbb{Z}^2}$ if $|U| = 7$, or $|U| = 4$ and $\rho \notin \{2,3,4\}$, up to a reordering of the directions (see [3], and also [1]). A uniqueness result in $\mathcal{C}_{\mathbb{Z}^2}$ holds true for a point X-rays tomography in a set P of 7 collinear sources, or 4 collinear sources if $\rho \notin \{2,3,4\}$. In these cases no lattice P-polygons exist, while, if the sources are not all collinear, no upper bound for $|P|$ is known to ensure a uniqueness result. However it can be shown that $|P| > 6$ ([2]), in contrast with $\mathbb{R}^2$, where it was proved (without using P-polygons, see [6]) that 4 radiographies suffices if the sources are no three collinear.

Presence of symmetry seems to be deeply related to the spread of ambiguities in the reconstruction processes. Thus, it is of interest to add knowledge about the geometric structure of such configurations. In [5] a characterization of switching component is given, as the linear combination of switching element. When U-polygons are considered the role of convexity does not appear immediately. However, an alternative geometric approach leads to a kind of convex counterpart of the above characterization

[1] Dulio P.- Peri C., *On the geometric structure of lattice U-polygons*, Discrete Math. (2007), to appear (doi: 10.1016/j.disc.2006.09.044)

[2] Dulio P.- Gardner R.J.- Peri C., *Discrete point X-rays*, SIAM J. Discrete Math. 20, no. 1 (2006), 171-188.

[3] R. J. Gardner and P. Gritzmann, Discrete tomography: Determination of finite sets by X-rays, *Trans. Amer. Math. Soc.* **349** (1997), 2271–2295.

[4] R.J. Gardner and P. McMullen, On Hammer's X-ray problem, *J. London Math. Soc.* (2) **21** (1980), 171-175.

[5]L. Hajdu and R. Tijdeman, Algebraic aspects of discrete tomography, *J. reine angew. Math* **534** (2001), 119-128.

[6] A. Volčič, A three-point solution to Hammer's X-ray problem, *J. London Math. Soc.* (2) **34** (1986), 349-359.

Nicola Fusco

Università di Napoli

Quantitative estimates for the isoperimetric inequality with the Gaussian measure

We give a new analytical proof of the isperimetric property of half-spaces with respect to the Gaussian measure. Moreover we show that if E is a set of finite perimeter with prescribed Gaussian volume, its distance from a half spaces can be estimated in a quantitative way by means of its Gaussian isoperimetric deficit.

Richard J. Gardner

Western Washington University, USA

Gaussian Brunn-Minkowski inequalities

(Joint work with Artem Zvavitch)

The talk is a report on joint work in progress with Artem Zvavitch. The focus is on inequalities of the Brunn-Minkowski type for Gauss measure γ_n in $\mathbb{R}^n$. The best-known of these are Ehrhard's inequality, which states that for $0 < t < 1$ and closed convex sets K and L in $\mathbb{R}^n$,

$$\Phi^{-1}\left(\gamma_n\left((1-t)K + tL\right)\right) \geq (1-t)\Phi^{-1}\left(\gamma_n(K)\right) + t\Phi^{-1}\left(\gamma_n(L)\right), \tag{1}$$

where $\Phi(x) = \gamma_1((-\infty, x))$, and the weaker inequality

$$\gamma_n\left((1-t)K + tL\right) \geq \gamma_n(K)^{1-t}\gamma_n(L)^t. \tag{2}$$

We obtain some results concerning other inequalities of this type, as well as a best-possible dual Gaussian Brunn-Minkowski inequality (where the Minkowski sum is replaced by radial sum).

Paolo Gronchi

Università degli studi di Firenze

Some remarks on the girth functions of convex bodies

Given a convex body $K \in \mathbb{R}^n$, the girth function $V_1(K|u^\perp)$ of K at $u \in S^{n-1}$ is the mean width of the orthogonal projection of K onto $u^\perp$. It is well known that $V_1(K|u^\perp)$ is the support function of a convex body $\Pi_1 K$, the projection body of order 1 of K.

We shall focus on necessary conditions for a support function to be the girth function of a convex body.

Christoph Haberl

Vienna University of Technology

Star body valued valuations

The concept of L_p centroid bodies is very useful in different situations within convexity. Recently, it also turned out that these bodies give rise to L_p analogues of the intersection body operator. We present characterization results of the L_p centroid body operator for $p \neq 0$ as star body valued valuation which has certain compatibility properties with the general linear group.

María A. Hernández Cifre

Otto-von-Guericke Universität Magdeburg
Universidad de Murcia

Bounds for the roots of the Steiner polynomial

(joint work with Martin Henk)

For two convex bodies $K, E \subset \mathbb{R}^n$ and a non-negative real number ρ the volume of $K + \rho E$ is a polynomial of degree n in ρ and it can be written as

$$V(K + \rho E) = \sum_{i=0}^{n} \binom{n}{i} W_i(K; E)\rho^i.$$

This polynomial is called the *relative Steiner polynomial* of K. The coefficients $W_i(K;E)$ are called the *relative quermassintegrals* of K, and they are just a special case of the more general defined *mixed volumes*. In particular, $W_0(K;E) = V(K)$ and $W_n(K;E) = V(E)$. If $E = B_n$, the Euclidean unit ball, the above polynomial becomes the classical *Steiner polynomial* and $W_i(K;B_n) = W_i(K)$ is the classical *i-th quermassintegral* of K.

In the planar case the relative inradius $r(K,E) = \sup\{r : \exists x \in \mathbb{R}^n \text{ with } x + r\,E \subseteq K\}$, circumradius $R(K,E) = 1/r(E,K)$ and quermassintegrals are related by the so called Bonnesen-type inequalities, e.g.

$$W_0(K;E) + 2W_1(K;E)\rho + W_2(K;E)\rho^2 \leq 0 \quad \text{if} \quad -R(K;E) \leq \rho \leq -r(K;E).$$

Here, the left-hand side is the 2-dimensional relative Steiner polynomial. In [2] B. Teissier studied Bonnesen-type inequalities in Algebraic Geometry and posed the problem of finding extensions of the properties derived from Bonnesen's inequality to higher dimensions. He was interested, in particular, in looking for bounds of the roots of the Steiner polynomial in terms of the relative in- and circumradius. In view of the properties in the planar case, in [1] it was conjectured that all convex bodies satisfy certain properties regarding the roots of the Steiner polynomial, and the inradius and the circumradius.

We have studied the location and the size of the roots of Steiner polynomials of convex bodies in the Minkowski relative geometry. We have shown that the roots of particular families of bodies fulfill the conjecture, but we also have found convex bodies violating each of the conjectured properties.

[1] J. R. Sangwine-Yager, Bonnesen-style inequalities for Minkowski relative geometry. *Trans. Amer. Math. Soc.* **307** (1) (1988), 373–382.

[2] B. Teissier, Bonnesen-type inequalities in algebraic geometry I. Introduction to the problem. *Seminar on Differential Geometry*, Princeton Univ. Press, Princeton, N. J., 1982, 85–105.

Daniel Hug

Universität Duisburg-Essen

Projection functions of convex bodies

The kth projection function of a convex body K assigns to any k-dimensional linear subspace of $\mathbb{R}^n$ the k-volume of the orthogonal projection of K to that subspace. If K is a ball, this function is constant. We discuss what is known about the converse problem of inferring sphericity of K from information about projection functions.

Markus Kiderlen

University of Aarhus

Geometric tomography: uniqueness, stability, and consistency under convexity assumptions

In geometric tomography one tries to retrieve information about an unknown subset K of $\mathbb{R}^d$ from certain measurements. These measurements are usually derived from K by geometrical operations like sections, projections and certain averages of them. If K is a convex body, convex geometric tools can be applied to obtain uniqueness, stability and consistency results. In addition, tomography induces new questions and developments in convex geometry. It is this interplay that will be presented in the talk. We will focus on three mayor issues:

1) **Analytical description of tomographic transforms.**
We restrict to tomographic measurements $F(K,\cdot)$ that can be written as (square integrable) function on the unit sphere S^{d-1}, and depend continuously and rotationally covariantly on K. Motivated by many examples, we require in addition that $F(K,\cdot)$ depends additively on an analytical representation $Q(K,\cdot)$ of K. As the analytical representation $Q(K,\cdot)$ may be a power of the support function, a power of the radial function, or a surface area measure, the class of tomographic data considered here is quite large. The first main result states that $F(K,\cdot)$ is a multiplier-rotation operator of $Q(K,\cdot)$, and can therefore be analyzed with tools from harmonic analysis. For $n \geq 3$, these operators are classical multiplier transforms.

2) **Stability results.**
We then turn to stability results stating that two convex bodies whose tomographic measurements are close to one another must be close in an appropriate metric on the family of convex bodies. We recall some classical stability results and improve the Hölder exponents for these transforms. The key idea for this improvement is to use the fact that support functions of convex bodies are elements of any spherical Sobolev space of derivative order less than 3/2. These results are illustrated by many examples ranging from classical projection and section functions to directed tomographic transforms.

3) **Consistency.**
In applications, only finitely many values of $F(K,\cdot)$ are available. Furthermore, measurements are typically disturbed by random measurement errors. If $\hat{K}_k$ is a least squares approximation of K based on k disturbed measurements of $F(K,\cdot)$ in directions $u_1,\ldots,u_k \in S^{d-1}$, the question arises if $\hat{K}_k$ converges almost surely to K as $k \to \infty$. It turns out that this strong consistency property holds, under weak assumptions on K, $u_1,\ldots,u_k$, $F(K,\cdot)$, and the distribution of the measurement errors. The proof of these results is based on stability estimates 2), entropy results for subsets of the family of convex bodies and tools from the theory of stochastic processes. It is applicable essentially to those transformations $F(K,\cdot)$, which preserve convexity (in the sense that for any convex body K the function $F(K,\cdot)$ is the support function of a convex body).

Bo'az Klartag

Princeton University

A central limit theorem for convex sets

Suppose X is a random vector, that is distributed uniformly in some n-dimensional convex set. It was conjectured that when the dimension n is very large, there exists a non-zero vector u, such that the distribution of the real random variable $< X,u >$ is close to the gaussian distribution. A well-understood situation, is when X is distributed uniformly over the n-dimensional cube. In this case, $< X,u >$ is approximately gaussian for, say, the vector $u = (1,...,1)/\sqrt{n}$, as follows from the classical central limit theorem. We prove the conjecture for a general convex set. Moreover, when the expectation of X is zero, and the covariance of X is the identity matrix, we show that for ï¿½most' unit vectors u, the random variable $< X,u >$ is distributed approximately according to the gaussian law. We argue that convexity - and perhaps geometry in general - may replace the role of independence in certain aspects of the phenomenon represented by the central limit theorem.

Marco Longinetti
Università degli Studi di Firenze

Convex Orbits and Orientations of a Moving Protein Domain

In [LSS] we study the facial structure and Carathéodory number of bodies which are the convex hulls of an orbit of the group of rotations acting on the space of anisotropic symmetric 3×3 tensors. This is motivated by the problem of determining the structure of some metalloproteins in aqueous solution.

Certain proteins consist of two rigid domains connected via a region which is flexible in aqueous solution (i.e. under physiological conditions). The problem which arises is to determine the relative position and orientation of these two domains. The proteins to which our study applies incorporate a paramagnetic metal ion in one rigid domain (these proteins include calmodulin). Interactions between the magnetic field of a dipole within the protein and the ion depend upon the dipole, the relative position of the ion and the dipole, and the magnetic susceptibility tensor of the ion. That part of the interaction coming solely from the relative orientation (via residual dipole coupling) may be inferred from nuclear magnetic resonance data. The residual dipole coupling between a paramagnetic metal ion and a dipole formed by atoms a and b within the protein depends upon the vector displacement r from the position of atom a to the position of atom b and the *magnetic susceptibility tensor* of the metal ion. It is a particular 3×3 symmetric matrix χ in a preferred coordinate frame determined by the tensor. The RDC interaction has the following vector formula

$$\delta \ := \ \frac{C}{\|r\|^5}\, r^T \chi r\,. \tag{1}$$

Here, C is a constant, r is expressed in the preferred coordinate system, and $\|r\| = \sqrt{r \cdot r}$ is the length of the vector r. A widely-studied case is that of calmodulin, which consists of two rigid domains, called the N-terminal and C-terminal domains, connected by a short flexible linker. The N and C-terminal domains are assumed to be rigid bodies whose structures are known in a reference frame called the lab frame.

We model the relative orientation of the two domains by a probability measure p on the group $SO(3)$ of rotations of $\mathbb{R}^3$. Incorporating data from multiple dipoles leads to the mean magnetic susceptibility tensor $\overline{\chi}$.

$$\overline{\chi} \ = \ \int_{SO_3} R^T \chi R \, dp(R)\,, \tag{2}$$

Recovering p from $\overline{\chi}$ is an ill-posed inverse problem. Nevertheless, $\overline{\chi}$ does contain useful information about p. [GLS] gave an algorithm that uses $\overline{\chi}$ to determine the maximum probability of a given relative orientation of the two domains. Since $\overline{\chi}$ lies in the convex hull of the orbit of χ under the group of rotations of $\mathbb{R}^3$, it admits a representation

$$(*) \qquad \overline{\chi} \ = \ \sum_j p_j R_j . \chi\,,$$

where the sum is finite, $\sum_j p_j = 1$ with $p_j \geq 0$, R_j is a rotation, and $R_j.\chi$ is the action of R_j on the tensor χ. If V_χ is the convex hull of this orbit of χ, then the minimal number of summands needed in $(*)$ to represent any point $\overline{\chi}$ in V_χ is the Carathéodory number of V_χ. The tensor $\overline{\chi}$ can be estimated from the RDC of several dipole pairs in the C-terminal domain. The experimental measures show that in terms of eigenvalue span, $\overline{\chi}$ is between 5 and 20 times smaller than χ. This indicates that p is not a point mass, i.e. the C-terminal domain moves with respect to the N-terminal domain.

It is known that the availability of N distinct mean susceptibility tensors $\overline{\chi}_k$ with respect to different metal ions $k = 1, \ldots, N$ increases the information about p for N up to 5, see [LLPS]. However, even the exact knowledge of 5 mean tensors $\overline{\chi}_k$ (i.e. 25 real numbers) does not allow the exact reconstruction of the probability measure p.

A possible approach to extract information from the mean tensors is to define $p_{\max}(R)$ as the maximal fraction of time that the C-terminal can stay in a particular orientation R, whilst still producing the measured mean tensors. The orientations with a large $p_{\max}$ have been shown to be consistent with what are thought to be the most favored orientations of the C-terminal. Looking for $p_{\max}$ implies the search for boundary points of the set of admissible data. Taking also into consideration the experimental error, the calculation of $p_{\max}$ is performed via a minimization procedure, using a discrete probability function p. The target function turns out very hilly, with many local minima. A fundamental question is the minimal number of orientations needed to reconstruct any admissible set of mean tensors $\overline{\chi}_k$. This is linked to the Carathéodory number of the set of admissible data. Motivated by the previous arguments and interesting from itself we investigate on the dimension , on the facial structure and on the Carathéodory number of the convex orbit $V_{1,\ldots,m}$ related to a given m-ple of anisotropic tensors $(\chi_1, \ldots, \chi_m)$. The main results for the one metal case, are complete. Let $0 \neq \chi \in W$ be an anisotropic tensor with maximal eigenvalue $M > 0$ and minimal eigenvalue $m < 0$, and let $\mu := -m - \frac{M}{2}$, and $\nu := M + \frac{m}{2}$.

Theorem 1. *The convex orbit V_χ has dimension five and it coincides with the space of symmetric 3×3 anisotropic tensors with eigenvalues in the interval $[m, M]$. When 0 is an eigenvalue of χ the convex orbit V_χ has Carathéodory number 2, otherwise 3.*

Theorem 2. *The boundary of V_χ is the union of a 2-dimensional family circles F_e^M, F_e^m called coaxial faces and parametrized by $e \in \mathbb{RP}^2$. The coaxial face F_e^M, and F_e^m, consists of tensors $\overline{\chi} \in V_\chi$ having $\langle e \rangle$ as an eigenvector with eigenvalue M, and m respectively; they are invariant under the action of the coaxial subgroup Q_e of $SO(3)$ and the intersection of any two coaxial faces lies in the orbit of $SO(3)$. Each face F_e^M is a circle of radius μ and each face F_e^m is a circle of radius ν. When χ has a repeated eigenvalue so that either μ or ν vanishes, then the corresponding coaxial face degenerates to a point. The faces F_e^M and F_e^m have a unique support hyperplane, unless they are degenerate.*

The main result for multiple tensors claim the following:

Theorem 3. *If d is the dimension of the linear span of the tensors $\chi_1, \ldots, \chi_m$ then the dimension of the convex orbit $V_{1,\ldots,m}$ is $5d$.*

The results about the facial structures for multiple tensors are not complete. In the two metal case (m=2) a 3-dimensional family of coaxial faces $F_{e,\alpha}$ is introduced; it is invariant under the action of the coaxial subgroup Q_e and parametrized by $\mathbb{RP}^2 \times S^1$, where $\langle e \rangle \in \mathbb{RP}^2$ and $\alpha \in S^1$. Our principal results for the facial structure of the convex orbit $V_{1,2}$ is

Theorem 4. *The union of coaxial faces $F_{e,\alpha}$ forms a 9-dimensional subset of the boundary of $V_{1,2}$ if and only if χ_1 and χ_2 have distinct eigenvectors. When this happens, all coaxial faces have dimension no more than 6 and they are facet, i.e maximal; almost all have dimension 6 and Carathéodory number 5. If χ_1 and χ_2 have a common eigenvectors, then the coaxial faces have dimension 2 or 4.*

[GLS] Gardner R J, Longinetti M, Sgheri L , *Reconstruction of orientations of a moving protein domain from paramagnetic data,* Inverse Problems, 2005, 879-898

[LLPS] Longinetti M, Luchinat C, Parigi G, Sgheri L, *Efficient determination of the most favoured orientations of protein domains from paramagnetic NMR data*, Inverse Problems, 2006, 1485-1502

[LSS] Longinetti M, Sgheri L , Sottile F, *On the the facial structure and Carathéodory number of Convex Orbit from RDC data*, 2007, preprint

Monika Ludwig

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Bivaluations on Convex Bodies

A *bivaluation* $\mu(K, L)$, defined for convex bodies K, L in $\mathbb{R}^n$, is a real valued function that is a valuation in either variable, provided that the other variable is held fixed. So, for example, for a bivaluation μ,

$$\mu(K_1, L) + \mu(K_2, L) = \mu(K_1 \cup K_2, L) + \mu(K_1 \cap K_2, L)$$

holds for convex bodies K_1, K_2, L if $K_1 \cup K_2$ is convex.

We show that if a bivaluation $\mu(K, L)$ is a Minkowski surface area (on the normed space with unit ball L), then there exists a constant c such that

$$\mu(K, L) = c\, V(K, \ldots, K, \Pi L^*) \quad \text{for all } K, L$$

that is, up to a constant, μ is the Holmes-Thompson surface area.

Mathieu MEYER

Université de Marne-La-Vallée

Increasing functions and Legendre tranforms

TBA

Emanuel Milman

The Weizmann Institute of Science, Rehovot

Generalized Intersection Bodies are Not Equivalent

The notion of an intersection-body was introduced by E. Lutwak in the 1970's, and has played an important role in the solution to the Busemann-Petty problem. Two different generalizations of this problem, considered by G. Zhang and A. Koldobsky, have given rise to two different generalizations of this notion. In 2000, Koldobsky asked whether these two classes of generalized intersection-bodies are in fact equivalent. We show that on one hand, these two classes share numerous identical structural properties, suggesting that the answer should be positive. On the other hand, we construct a counter-example, which provides a surprising negative answer to this question in a strong sense. This implies the existence of non-trivial non-negative functions in the range of the spherical Radon transform, and the existence of non-trivial spaces which embed in L_p for certain negative values of p.

Vladimir I. Oliker

Emory University, Atlanta

Convexity, optimal mass transport and design of mirrors

The purpose of this talk is to survey the results and methods on the Reflector Problem, which is a very natural generalization of the classical Minkowski problem. In recent years this problem attracted much attention of mathematicians. In the simplest case, a reflector is a convex hypersurface which is the boundary (or a portion of it) of a compact convex body obtained by taking the intersection of a family of (solid) confocal paraboloids of revolution. Reflectors arise naturally in geometrical optics and are used in design of light reflectors and reflector antennas. The reflector problem consists in determining a mirror which reflects a bundle of light rays from a point source with a given intensity distribution into an output set of rays illuminating a prespecified region of a sphere with prescribed intensity. In analytic formulation this problem requires a solution of a problem with features appearing in many nonlinear geometric and geometrical optics problems as well as in problems in optimal mass transportation theory. The classical convexity methods provide a natural framework for studying this problem and its solution uses methods from several areas of mathematics.

Grigoris Paouris

Université de Marne-la-Vallée

Inequalities for the negative moments of the Euclidean norm on a convex body

Let K be a convex body in $\mathbb{R}^n$ of volume 1 and the center of mass at the origin. Define, for $p > -n$

$$I_p(K) = \left(\int_K \|x\|_2^p dx \right)^{1/p}$$

and

$$q_*(K) = \sup\{q > 0 : k_*(Z_q^\circ(K)) \geq q\},$$

where $k_*(Z_q^\circ(K))$ are the Dvoretzky numbers of the L_q-centroid bodies of K. It was recently proved that $I_p(K) \leq cI_2(K)$ for all $p \in (2, q_*(K)]$. In this talk we will present the extension of this result to the case of negative p. We will show that there exists $c > 0$ such that

$$I_p(K) \leq cI_{-p}(K) \text{ for all } p \in (0, q_*(K)].$$

Carla Peri

Università Cattolica, Milano

Hyperplane sections of convex bodies

We prove sharp inequalities for the areas of hyperplane sections bisecting the volume of a convex body. This leads to a relative isoperimetric inequality for arbitrary hyperplane sections.

Shlomo Reisner

Haifa University

Hausdorff Approximation of 3D convex Polytopes

(joint work with Mario Lopez)

Let P be a convex polytope in $\mathbb{R}^3$ (or a convex polygon in $\mathbb{R}^2$), having n vertices. Given a positive integer $k < n$ (k must be big enough but there is no constraint on the relative size of k with respect to n) we present a linear (in n) time algorithm that selects k of the vertices of P, in such a way that for the convex hull Q of the selected vertices we have

$$d_H(P,Q) \le \frac{cR}{k} \qquad \left(\frac{cR}{k^2} \text{ in the 2D case}\right), \tag{1}$$

where c is a fixed constant and R is the minimal radius of a ball containing P. d_H denotes the Hausdorff distance. The significance of the estimate (1) is in the fact that, up to the value of the constant c, this is the best possible worst-case estimate. Analogous algorithm can be presented where the role of vertices is taken by facets. Special properties of 3 dimensional (and 2 dimensional) polytopes are used in developing a polynomial (linear) time algorithm. In higher dimensions we can only show that under similar assumptions, there *exists* a selection of k vertices of P such that their convex hull Q satisfies

$$d_H(P,Q) \le \frac{cR}{k^{2/(d-1)}} \, . \tag{2}$$

But we have no constructive way to select these vertices (the estimate (2) is again best possible, the fact that it can be achieved using vertices of P itself, which is well known for the symmetric distance, seems to be new for the Hausdorff distance).

Matthias Reitzner

TU Vienna

A Classification of SL(n) invariant Valuations

(joint work with Monika Ludwig)

Let $\mathcal{K}_0^n$ denote the space of convex bodies that contain the origin in their interiors. A functional $\Phi : \mathcal{K}_0^n \to \mathbb{R}$ that satisfies the inclusion-exclusion relation

$$\Phi(K) + \Phi(L) = \Phi(K \cup L) + \Phi(K \cap L)$$

whenever $K, L, K \cup L, K \cap L \in \mathcal{K}_0^n$, is called a *valuation*. A valuation is upper semicontinuous, if

$$\Phi(K) \ge \limsup_{n\to\infty} \Phi(K_n)$$

for every K and every sequence K_n with $K_n \to K$. It is SL(n) invariant, if $\Phi(AK) = \Phi(K)$ for every $A \in \mathrm{SL}(n)$ holds.

A classification of upper semicontinuous and SL(n) invariant valuations is established. As a consequence, characterizations of centro-affine and L_p affine surface areas are obtained.

Mark Rudelson

University of Missouri, Columbia

Almost spherical sections of a cross-polytope generated by random matrices

A celebrated theorem of Kashin states that a random section of the cross-polytope in $\mathbb{R}^n$ of dimension $m \sim n$ is close to the section of the inscribed ball. Since the deterministic constructions of such sections for m close to n are still unknown, the attention was recently attracted to simple probabilistic methods. One of such constructions, which is probably easiest to generate, is a subspace of $\mathbb{R}^n$ spanned by m vectors with independent random ± 1 coordinates. We prove that, with high probability, for any $m < n$ the section of the n-dimensional cross-polytope by this subspace will be close to a ball. Moreover, the distance to a ball can be bounded by a power of the relative codimension $(n-m)/n$. The proof combines ideas from convex geometry and random matrix theory.

Paolo Salani

Università degli studi di Firenze

Serrin type overdetermined problems for Hessian equations

(joint work with B. Brandolini - C. Nitsch - C. Trombetti)

We prove the simmetry of solutions to overdetermined problems for a class of fully nonlinear equations, namely the Hessian equations.

Pecisely, we consider the following class of overdetermined problems

$$\begin{cases} S_k(D^2u) = \binom{n}{k} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \qquad k \in \{1, \dots, n\} \\ \dfrac{\partial u}{\partial \nu_x} = 1 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded, open, connected domain of $\mathbb{R}^n$ whose boundary is of class C^2, with outer unit normal ν_x and $S_k(D^2u)$ is the k-th elementary symmetric function of the eigenvalues of D^2u.
Notice that, when $k = 1$ we get back to Poisson equation, while $k = n$ carries out the well-known Monge-Ampère equation.

We prove that, if a solution $u \in C^2(\overline{\Omega})$ of (1) exists, then $u = \frac{|x|^2-1}{2}$ up to a translation and Ω is the unitary ball.

In case of Poisson equation, our proof is alternative to the ones proposed by Serrin (moving planes) and by Weinberger, and it makes no direct use of the maximum principle, while it enlightens a relation between Serrin problem and isoperimetric inequality.

In the talk, I will mainly deal with the Monge-Ampère case, showing a proof based on elementary tools from convex analysis and also proving the stability of radial symmetry for this problem.

Eugenia Saorín

Universidad de Murcia

On a problem by Hadwiger

(joint work with María A. Hernández Cifre)

Let K be a convex body in the Euclidean space and ρ be a positive real number. The *outer* and the *inner parallel bodies of* K *at distance* ρ are defined, respectively, as the Minkowski sum $K+\rho\mathbb{B}^n$ and the *Minkowski difference* $K\sim\rho\mathbb{B}^n=\left\{x\in\mathbb{R}^n : \rho\mathbb{B}^n+x\subset K\right\}$.

In 1955, H. Hadwiger studied a problem related with the differentiability of the classical functionals volume V, surface area S and integral mean curvature M, in the following sense: he considered the so called *full system of parallel bodies* of a convex body K, this is, the one-parameter-depending family

$$K_\rho := \begin{cases} K\sim(-\rho)\mathbb{B}^n & \text{for } -r(K)\le\rho\le 0,\\ K+\rho\mathbb{B}^n & \text{for } 0\le\rho<\infty,\end{cases}$$

where $r(K)$ denotes the inradius of K, and he studied the differentiability of the functionals V, S and M with respect to the full system of parallel bodies. More precisely, since all convex bodies verify $'V(\rho)=V'(\rho)=S(\rho)$, he asked for a characterization of the convex bodies satisfying the relations $'S(\rho)=S'(\rho)=2M(\rho)$ and/or $'M(\rho)=M'(\rho)=4\pi$.

We have studied this problem in $\mathbb{R}^n$ and also its connection with some questions on the so called "alternating Steiner polynomial".

Rolf Schneider

Universität Freiburg

Characterizations of duality

Answering a question of Vitali Milman, we characterize the duality of convex bodies in the following way. Let $\mathcal{K}^d_{(0)}$ be the space of compact convex sets in Euclidean space $\mathbb{R}^d$ having 0 as interior point ($d\ge 2$). If $\psi:\mathcal{K}^d_{(0)}\to\mathcal{K}^d_{(0)}$ is a mapping such that $\psi\circ\psi=id$ and $\psi(K\cap L)=\mathrm{conv}(\psi(K)\cup\psi(L))$ for all $K,L\in\mathcal{K}^d_{(0)}$, then $\psi(K)=gK^*$ for all $K\in\mathcal{K}^d_{(0)}$, where g is a fixed (selfadjoint) linear transformation of $\mathbb{R}^d$ and K^* denotes the polar body of K. We deduce this result from a classification of the endomorphisms of the lattice $(\mathcal{K}^d_{(0)},\cap,\vee)$, where $K\vee L:=\mathrm{conv}(K\cup L)$. For the proof of the latter, we profit from ideas of Peter Gruber (1991, 1992), who has determined the endomorphisms of two other lattices of convex sets. (Joint work with Károly Böröczky Jr.)

Similar results are obtained for the lattice $(\mathcal{C}^d,\cap,+)$ of closed convex cones in $\mathbb{R}^d$, for $d\ge 3$.

Franz Schuster

TU Vienna

Rotation Invariant Minkowski Classes of Convex Bodies

(joint work with Rolf Schneider)

A Minkowski class is a closed subset of the space of convex bodies in Euclidean space $\mathbb{R}^n$ which is closed under Minkowski addition and non-negative dilatations. A convex body in $\mathbb{R}^n$ is universal if the expansion of its support function in spherical harmonics contains non-zero harmonics of all orders. If K is universal, then a dense class of convex bodies M has the following property. There exist convex bodies T_1,T_2 such that $M+T_1=T_2$, and T_1,T_2 belong to the rotation invariant Minkowski class generated by K. We present a recent result which shows that every convex body K which is not centrally symmetric has a linear image, arbitrarily close to K, which is universal. A modified version of the result holds for centrally symmetric convex bodies. In this way, we strengthen a result of S. Alesker, and at the same time give a more elementary proof.

Carsten Schütt

Universität Kiel

Simplices in the Euclidean ball

We prove the following result:

If $\varrho > 0$ and $S = \mathrm{conv}\{x_1, \ldots, x_{n+1}\}$ is an n–simplex in $\mathbb{R}^n$ with $\|x_i\| \geq \varrho$, $i = 1, \ldots, n+1$, then

$$\int_S \|x\|^2 dx \geq \frac{\varrho^2}{9n} \cdot V(S).$$

Cristina Trombetti

Università degli Studi di Napoli

A quantitative version of Polya-Szego principle

The operation of radially decreasing symmetrization is well known not to increase Dirichlet type integrals of Sobolev functions. We estimate the deviation of a function from its symmetral in terms of the gap between their Dirichlet integrals.

Elisabeth Werner

Case Western Reserve University, Cleveland

On L_p-affine surface areas

Let K be a convex body in $\mathbb{R}^n$ with centroid at 0 and B be the Euclidean unit ball in $\mathbb{R}^n$ centered at 0. We show that

$$\lim_{t\to 0} \frac{|K| - |K_t|}{|B| - |B_t|} = \frac{O_p(K)}{O_p(B)},$$

where $|K|$ respectively $|B|$ denotes the volume of K respectively B, $O_p(K)$ respectively $O_p(B)$ is the p-affine surface area of K respectively B and $\{K_t\}_{t\geq 0}$, $\{B_t\}_{t\geq 0}$ are general families of convex bodies constructed from K, B satisfying certain conditions.

Deane Yang

Polytechnic University, Brooklyn, New York

Generalizations of the John ellipsoid

The classical John ellipsoid is the ellipsoid of maximal volume contained inside a convex body. It can be viewed as arising naturally in the L_∞ Brunn-Minkowski theory. An ellipsoid that is in some sense dual to the John ellipsoid is the ellipsoid of minimal volume containing the convex body; this ellipsoid belongs to the L_∞ dual Brunn-Minkowski theory. Lutwak, Yang, and Zhang showed that the notion of L_p mixed volume can be used to extend the definition of the John ellipsoid to L_p John ellipsoids. Bastero and Romance did the same independently and also used the notion of L_p dual mixed volume to extend the definition of the dual John ellipsoid to dual L_p John ellipsoids.

Lutwak, Yang, and Zhang have also extended all of this from convex bodies to random vectors, where the dual L_p John ellipsoid corresponds to an L_p moment matrix, and the L_p John ellipsoid to an L_p Fisher information matrix, generalizing the well-known L_2 case. These information measures satisfy sharp information theoretic inequalities extending classical L_2 information theoretic inequalities.

Vladyslav Yaskin

University of Oklahoma, Norman

Modified Shephard's problem on projections of convex bodies

We disprove a conjecture of A. Koldobsky asking whether it is enough to compare $(n-2)$-derivatives of the projection functions of two symmetric convex bodies in the Shephard problem in order to get a positive answer in all dimensions.

Maryna Yaskina

University of Oklahoma, Norman

Shadow Boundaries and the Fourier Transform

(joint work with Paul Goodey and Vladyslav Yaskin)

We investigate the Fourier transform of homogeneous functions on $\mathbb{R}^n$ which are not necessarily even. These techniques are applied to the study of nonsymmetric convex bodies, in particular to the question of reconstructing convex bodies from the information about their shadow boundaries.

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