Research Training Network “Phenomena in High Dimension”

EDUCATIONAL WORKSHOP ON GEOMETRIC INEQUALITIES

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Dipartimento di Matematica - Università della Calabria, Dipartimento di Matematica “U. Dini”- Università di Firenze

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ABSTRACTS OF MAIN LECTURES

Richard. J. GARDNER *(Western Washington Univ.,U.S.A)*

The dual Brunn-Minkowski theory and some of its inequalities

As the title suggests, the focus of the talk is the dual Brunn-Minkowski theory, initiated by Erwin Lutwak in 1975, and developed since by him and others. First, an elementary introduction is provided to the basics, dual mixed volumes of star bodies, the dual Aleksandrov-Fenchel inequality and some of its consequences. A recent partial extension to bounded Borel sets with a motivating application in local stereology is discussed. After this, intersection bodies are defined and the Busemann intersection inequality is stated in a general form. Finally, affine and dual affine quermassintegrals are introduced and some known and conjectured inequalities involving them are examined.

Paul GOODEY *(Univ. of Oklahoma, U.S.A)*

Classes of centrally symmetric convex bodies

Zonoids are highly symmetric convex bodies which arise in many geometric settings. They can be characterized by integral representations of their support functions or in terms of projection bodies. Both these characterizations make use of the lengths of the projections of the body onto lines. We will consider various classes of centrally symmetric bodies that arise from generalizing these characterizations of zonoids to the setting of higher dimensional projections. This will give rise to a number of different generalizations of zonoids. In this talk, we will investigate the connections between these different generalizations.

Peter M. GRUBER *(Technische Univ. Wien, Austria)*

John type theorems

Using an idea of Voronoi we prove several results characterizing ellipsoids which contain a given convex body and are minimal in some sense.

Daniel HUG *(Albert Ludwigs Univ. Freiburg, Germany)*

Valuations, integral geometry and linear dependencies

Joint work with R. Schneider and R. Schuster

A classical result of integral geometry is the principal kinematic formula, attributed to Blaschke, Santaló and Chern. Another important result, due to Hadwiger, is the characterization theorem for motion invariant, continuous valuations on the space of convex bodies. Both theorems are connected via Hadwiger's integral geometric formula for general functionals. More recently, a paper by Peter McMullen from 1997 stimulated the investigation of the interplay between tensor valuations and integral geometry. In this direction, a first culmination point was the classification of isometry covariant, continuous valuations by Semyon Alesker. This theorem provides a far reaching generalization of Hadwiger's theorem on valuations and of the special case concerning vector valuations, which had been considered previously.

In this talk, we will start with a brief outline of these developments and then delve into integral geometry of tensor valuations. Some special Crofton and kinematic formulas as well as applications to stochastic geometry have been found. Here we aim at describing a more complete picture of available results and underlying methods. In a final part, we consider the problem of determining all linear dependencies among the basic isometry covariant functionals, a question originally raised by Peter McMullen.

Peter McMULLEN *(Univ. College London, England)*

The algebra of polyhedra

At the heart of much of the metrical theory of convex bodies and related topics, as well as some of the combinatorial theory of polytopes, lies the concept of a valuation. For polytopes, it turns out to be very useful to develop the initial part of the theory of valuations at the more general level of polyhedra (polyhedral sets). There is then, in fact, a ring structure, which is probably well known to this audience. The addition on the *polyhedron ring* Γ is induced by the valuation relation:

$$[P \cup Q] + [P \cap Q] = [P] + [Q]$$

whenever P, Q are polyhedra such that $P \cup Q$ is convex. The multiplication on $\varGamma$ is induced by Minkowski addition:

$$[P] \cdot [Q] = [P + Q].$$

There is then a range of structure theorems on $\varGamma$, among which one will recognize generalized angle-sum relations. Of course, all these carry over to the polytope ring $\varPi$, where the implications are perhaps more familiar.

What is less familiar is the co-ring structure on $\varGamma$. Roughly speaking, the laws of a co-ring dualize those of a ring, if one writes them down very formally. Thus, for example, multiplication is a mapping $\mu\colon \varGamma \otimes \varGamma \to \varGamma$, with certain additional properties (associativity – the distributive laws are taken care of by the formulation). Then co-multiplication will be a certain mapping $\kappa\colon \varGamma \to \varGamma \otimes \varGamma$ (with an appropriate definition of co-associativity). Indeed, with an appropriate choice of κ, there will be the structure of a bi-ring on $\varGamma$, meaning that the ring and co-ring operations are compatible. While this may seem hopelessly abstract, in fact it will be shown that some well-known valuations – and many variants of them – arise naturally through the co-ring structure.

Vitali D. MILMAN *(Univ. Tel Aviv, Israel)*

The ZigZag approximation of the euclidean ball and other applications to Convex Geometry of Chernoff probabilistic bound

Joint work with S. Artstein and O. Friedland

The aim of the talk is to demonstrate that the well known Chernoff estimates from Probability theory can be used in an "asymptotic" geometric context for a very broad spectrum of problems and lead to new and improved results. We will briefly describe Chernoff bounds, and then discuss the motivation for so called "ZigZag approximation" for balls, sign-embedding of euclidean spaces to l_1 spaces and, perhaps, some other problems of asymptotic convexity.

Alain PAJOR *(Univ. Marne-la-Vallée, France)*

Geometry of random $\{-1, 1\}$-polytopes

Random $\{-1, 1\}$-polytopes demonstrate extremal behavior with respect to many geometric characteristics. We illustrate this by showing that the combinatorial dimension, entropy and mixed volumes of these polytopes are extremal at every scale of their arguments.

Aljoša VOLČIČ *(Univ. della Calabria, Italy)*

Hammer's X ray problem, open questions and algorithmic aspects

More than forty year ago P.C. Hammer asked his famous question about determination of convex bodies from parallel and point X-rays.

Beginning in the eighties several answers appeared to the original question and to some natural extensions, building up a small but nice theory. Many results concerned determination in the sense of uniqueness, fewer in the sense of actual reconstruction.

This talk will offer an overview on the subject paying special attention to open problems and to reconstruction algorithms.

Wolfgang WEIL *(Univ. Karlsruhe, Germany)*

Determination of convex bodies by projection functions

A centrally symmetric n-dimensional convex body $K \subset \mathbb{R}^n$ is uniquely determined (up to translation) by any of its projection functions. This classical result has many variants and extensions, some of which are presented in this talk.

Jörg M. WILLS *(Univ. Siegen, Germany)*

Convex Bodies and Lattice Points

The number of lattice points in a convex body is a central topic in Geometry of Numbers since Gauss, Dirichlet, Hermite, Minkowski and Blichfeldt. Here we investigate relations and inequalities between the Ehrhart polynomial card$(\lambda P \cap \mathbb{Z}^d)$, ($P$ lattice polytope), its coefficients, its zeros and two closely related polynomials, which are built up of intrinsic volumes and of Minkowski's successive minima. The motivation is to get more insight into the (convex-) geometric and algebraic properties of the Ehrhart polynomial. Old and new results and some open problems are presented.

ABSTRACTS OF SHORT COMMUNICATIONS

Jesús M. ALDAZ *(Univ. de La Rioja, Spain)*

The Hardy-Littlewood maximal operator in high dimensions

The behaviour of the Hardy-Littlewood maximal operator in high dimensions has been studied by several authors, among others E. M. Stein, J. Bourgain and A. Carbery. They obtained bounds for the operator that are independent of the dimension. We describe what is known in the area, as well as some open problems.

Gennadiy AVERKOV *(Technische Univ. Chemnitz, Germany)*

On inequalities for convex bodies and the geometry of linear normed spaces

Many results from convexity can be reformulated in terms of Minkowski geometry (i.e. the geometry of finite dimensional linear normed spaces). On the other hand, Minkowskian tools can be used for solving problems from convex geometry that at the first glance are not related to Minkowski spaces. This concerns also geometric inequalities and optimization problems for convex bodies in Euclidean spaces, some of which can be resolved with the help of Minkowski geometry.

René BRANDENBERG *(Technische Univ. München, Germany)*

A geometric inequality on volume minimal ellipsoids

In *Ellipsoids of maximal volume in convex bodies* Ball could show a geometric inequality between the (j-dimensional) volume maximal ellipsoid contained in the intersection of some convex body C and a j-flat F and the (j-dimensional) volume of the intersection of the (fulldimensional) volume maximal ellipsoid contained in C itself and F. Also he could show that a sharper bound holds if C has to be 0-symmetric. We give a similar result on volume minimal enclosing ellipsoids and show that the bound stays even tight for symmetric bodies. Afterwards we give some applications of the inequality.

Maria A. HERNÁNDEZ CIFRE *(Univ. de Murcia, Spain)*

The Steiner Polynomial and its consequences on the Blaschke diagram

Joint work with E. Saorín

Let K be a compact convex set in the euclidean space. It is well known that the volume of the outer parallel body $K+\rho\mathbb{B}^n$ can be expressed as a polynomial of degree the dimension n in the parameter ρ,

$$V(K+\rho\mathbb{B}^n)=\sum_{i=0}^{n}\binom{n}{i}W_i(K)\rho^i,$$

which is known as the *Steiner polynomial* of the body K. In some works by Teissier, Oda, Sangwine-Yager and others, the *alternating Steiner polynomial* has been studied (i.e., the above polynomial in which the variable ρ is changed by $-\rho$). It is also a natural question to study the roots of Steiner's polynomial itself, and to try to find out whether they have any geometric meaning.

On the other hand, Blaschke (1916) considered a compact convex set K in the euclidean 3-space $\mathbb{R}^3$, with volume $V=V(K)$, surface area $S=S(K)$, and integral of the mean curvature $M=M(K)$. He asked for a characterization of the set of all points in $\mathbb{R}^3$ of the form $\big(V(K),S(K),M(K)\big)$ as K ranges over the family of all compact convex sets in $\mathbb{R}^3$, or, equivalently, for a characterization of the set of all points (x,y) in $\mathbb{R}^2$ of the form

$$x=\frac{4\pi S}{M^2}\quad\text{and}\quad y=\frac{48\pi^2 V}{M^3}.$$

The latter set is called the *Blaschke diagram*.

In this talk we intend to analyze the relationship between these two problems: first, studying the roots of the three dimensional Steiner polynomial from a geometric point of view, in the sense of characterizing convex bodies according to the type of roots of its Steiner polynomial; then, showing that there is a close relationship with the famous Blaschke problem, for which we can obtain interesting consequences.

Nico DÜVELMEYER *(Technische Univ. Chemnitz, Germany)*

On characterizations of Euclidean spaces

One possible extension of Euclidean geometry is the geometry of (Banach-)Minkowski spaces, i.e., of finite dimensional real linear normed spaces. All properties of affine geometry remain valid, but geometric terms like angles or area do not have straightforward analogues in this context. Furthermore, there are even different ways of generalizing these geometric terms. The study of the relations between these terms often involves special properties of the considered Minkowski space. One interesting property, of course, is to be isometric to a Euclidean space. In this direction various characterizations are known, but the discovery of new ones is still an interesting field of investigation. In this lecture, such a new characterization will be presented.

Fausto FERRARI *(Univ. di Bologna, Italy)*

Distance function, metric normal and sets of positive reach in the Heisenberg group

In the Euclidean setting the distance function from a set $\mathcal{S}$ contains many information about the set $\mathcal{S}$ itself. In particular, for any given point P on the regular surface $\mathcal{S}$, the distance from $\mathcal{S}$ of all points on the normal straight line starting from P equals, in a neighborhood of the point P, the distance from P itself. Notice that smoothness of the distance function, with respect to the regularity of the set $\mathcal{S}$, can be deduced starting from such remark, see e.g. [4], [5].

In a joint work with Nicola Arcozzi, from University of Bologna, see [1] and [2], we defined and studied in the Heisenberg group $\mathbb{H}$ the notion of *metric normal*. For any given set $\mathcal{S} \subset \mathbb{H}$ and for any point $P \in \mathcal{S}$, the metric normal of the set $\mathcal{S}$ in P is the set

$$\mathcal{N}_P S = \{Q \in \mathbb{H} : \inf_{R \in \mathcal{S}} d(Q,R) = d(Q,P)\},$$

where d is the Carnot-Charathéodory distance in the Heisenberg group.

Since the notion of metric normal is related to that one of *positive reach* (introduced by Federer), see [3], it seems quite natural to apply the results we deduced on the metric normal to investigate the existence of a sort of Steiner formula for sets of positive reach in the Heisenberg group, (see [3] for the Euclidean setting), and maybe understand well what a convex set is in this framework.

As a first step in such direction, we gave sufficient conditions about the existence, far from characteristic points, of the metric normal of sufficiently smooth surfaces $\mathcal{S}$.

In particular we proved that there exist smooth surfaces $\mathcal{S}$ such that their own horizontal Hessian matrices of the distance function with sign from $\mathcal{S}$ are not bounded while, on the contrary, in Euclidean setting they are. Moreover we proved that the Kohn-Laplace operator of the distance function with sign from $\mathcal{S}$, evaluated in the non characteristic points of the surface, is proportional to the intrinsic divergence of the intrinsic normal vector field to the smooth surface itself, namely the mean curvature of the surface $\mathcal{S}$ itself.

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[1] N. Arcozzi, F. Ferrari, *Metric normal and distance function in the Heisenberg group*, preprint (2005).
[2] N. Arcozzi, F. Ferrari, *The Hessian of the distance from a surface and Mean Curvature in the Heisenberg group*, preprint (2004).
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[4] D. Gilbarg;N.S. Trudinger, *Elliptic partial differential equations of second order*. Second edition. Springer-Verlag, Berlin, 1983.
[5] S. G. Krantz; H. R. Parks, *Distance to C^k hypersurfaces*, J. Differential Equations 40 (1981), no. 1, 116–120.

Antonio GRECO *(Univ. di Cagliari, Italy)*

Weak convexity implies strict convexity via maximum principle on manifold

This talk deals with the problem of convexity of the level sets for solutions u to elliptic semilinear equations in a convex ring attaining constant values on the boundary.

Under convenient assumptions on the equation, the quasi-concavity function $Q(x,y) = u((x+y)/2) - \min(u(x), u(y))$ satisfies an elliptic inequality on the manifold $\{(x,y) : u(x) = u(y)\}$.

This implies that if the level sets of u are convex then they are strictly convex.

The result applies to anisotropic equations such as, for instance, $\Delta u = |u_1|$, where u_1 is the partial derivative of u with respect to the variable x_1.

Paolo GRONCHI *(Univ. di Firenze, Italy)*

Petty's projection inequality

Joint work with S. Campi

We present a proof of Petty's inequality on the volume of the polar projection body of a convex body which involves centroids bodies and shadow systems.

Alexander KOLDOBSKY *(Univ. of Missouri, U.S.A.)*

Banach subspaces of $L_p, p < 1$

Joint work with N.Kalton

For $0 < p < 1$ we give examples of Banach spaces isometrically embedding into L_p but not into any L_r with $p < r \leq 1$.

Kurt LEICHTWEISS *(Univ. Stuttgart, Germany)*

Non-euclidean convex geometry

It is not too much known about non-euclidean convex geometry with the exception of symmetrization in order to get the isoperinetric inequality, introduced by E.Schmidt (see Burago-Zalgaller: "Geometric Inequalities" §9 and §10). The two things which one misses at first in this context are Minkowski's addition and a suitable support function. We want to give a short survey of our results to fill these gaps in the planar case with applications to mixed volumes and curves of constant width.

Francesco LEONETTI *(Univ. di L'Aquila, Italy)*

Dividing a set into convex subsets

We consider a set $E \subset \mathbb{R}^n$ and we try to divide it into a finite number of subsets $E = \cup_{i=1}^k E_i$ where E_i is convex with nonempty interior. For example, we take the "8" picture, that is, two disks, one upon the other, touching at one point: we can decompose this "8" picture into two convex pieces, the two disks. We can also split the two disks thus obtaining four convex subsets. This means that we may have different decompositions; moreover, the number of subsets in a decomposition may differ from the number of subsets in another one. Thus we are led to consider *minimal* decompositions, that is, those with least number of subsets and we try to estimate this least number from below. In order to do that, we recall the following theorem concerning the $n-1$ Hausdorff measure of the boundary ∂A of a bounded, convex, open set $A \subset \mathbb{R}^n$, see [1].

Theorem 1. *If $A \subset B \subset \mathbb{R}^n$ with A open and convex, B bounded, then*

$$\mathcal{H}^{n-1}(\partial A) \leq \mathcal{H}^{n-1}(\partial B) \tag{1}$$

The previous theorem says that, when you start from a bounded, open, convex set A and you enlarge it a little bit, then the measure of the boundary ∂A increases too. This is false, in general, if you start from a nonconvex set. However, if the nonconvex set can be decomposed into a finite number of convex subsets, then the following result holds true, [2]

Theorem 2. *For every $k \in \mathbb{N}$ there exists $c(k) \in (0, k]$ such that, for every $E \subset \mathbb{R}^n$ satisfying*

$$E = \cup_{i=1}^k E_i \tag{2}$$

$$E_i \text{ convex} \qquad \forall i = 1, \ldots, k \tag{3}$$

and

$$\text{interior of } E_i \neq \emptyset \qquad \forall i = 1, \ldots, k \tag{4}$$

for every bounded $B \subset \mathbb{R}^n$ verifying

$$E \subset B \tag{5}$$

it turns out that

$$\mathcal{H}^{n-1}(\partial E) \leq c(k)\mathcal{H}^{n-1}(\partial B) \tag{6}$$

In [2] we show that for every $k \in \mathbb{N}$ for every $\epsilon \in (0, 1)$ there exists E verifying (2), (3), (4) and there exists B bounded, convex and satisfying (5) such that

$$0 < (k - \epsilon)\mathcal{H}^{n-1}(\partial B) \leq \mathcal{H}^{n-1}(\partial E) < +\infty \tag{7}$$

thus

$$c(k) = k \tag{8}$$

Take $\epsilon = 1/2$ in (7), then

$$(k - 1/2) \leq \frac{\mathcal{H}^{n-1}(\partial E)}{\mathcal{H}^{n-1}(\partial B)} \leq k \tag{9}$$

so that, taking the supremum upon B bounded and convex, with $E \subset B$, we get

$$(k - 1/2) \leq \sup_B \frac{\mathcal{H}^{n-1}(\partial E)}{\mathcal{H}^{n-1}(\partial B)} \leq k \tag{10}$$

eventually, taking the upper integer part, we arrive at

$$\text{upper integer part of } \left(\sup_B \frac{\mathcal{H}^{n-1}(\partial E)}{\mathcal{H}^{n-1}(\partial B)}\right) = k \tag{11}$$

This means that, for the above mentioned E's taken from [2], the decomposition $E = \cup_{i=1}^{k} E_i$ is minimal with respect to the number k of convex pieces E_i with not empty interior.

References

[1] G. Buttazzo - V. Ferone - B. Kawohl *Minimum problems over sets of concave functions and related questions*, Math. Nachr., **173** (1995), 71-89.

[2] F. Leonetti *Convex components of a set and the measure of its boundary*, to appear.

Marco LONGINETTI *(Univ. di Firenze, Italy)*

Affinely regular polygons as maximizers of area functionals

Joint work with P. Gronchi

Given a convex polygon P with n vertices z_j, we define $W_j(P)$ to be the triangle $z_{j-1}z_jz_{j+1}$, and $T_j(P)$ to be the (possibly infinite) triangle outside P bounded by a side z_jz_{j+1} and the continuations of its contiguous sides. Define the following area functionals on the class $\mathcal{P}_n$ of plane convex polygons with *exactly* n vertices:

$$F(P) = \min_{j=1,\dots,n} \frac{|W_j(P)|}{|P|}\,, \quad G(P) = \min_{j=1,\dots,n} \frac{|T_j(P)|}{|P|}\ .$$

We prove that *the maximizers of F and G are the affinely regular polygons with n vertices.*

In previous papers the author showed the connection of the second functional above with the stability of the so called Hammer's X-ray problem for homogeneous plane convex bodies. These functionals are also connected with the affine isoperimetric inequality and with inner (or outer) approximation of a given convex n-gon with $(n-1)$-gons.

Henceforth, we say that a polygon P has the *inner (outer) equivalent triangle* property when P belongs to the following classes (respectively)

$$\Phi_n = \{P \in \mathcal{P}_n : |W_1(P)| = |W_2(P)| = \cdots = |W_n(P)|\}\,,$$

$$\Gamma_n = \{P \in \mathcal{P}_n : |T_1(P)| = |T_2(P)| = \cdots = |T_n(P)|\}.$$

A second significant property of maximizers is the following. Let l_j be the length of the side z_jz_{j+1} of P, and d_j the length of the diagonal z_{-1}, z_{j+2}. We define I_n as the class of convex polygons in $\mathcal{P}_n$ satisfying the *diagonal-side* property, i.e.

$$\frac{d_{j+1} - l_{j+1}}{d_{j+1}} \frac{d_j - l_j}{l_j} = \frac{d_{j-2} - l_{j-2}}{d_{j-2}} \frac{d_{j-1} - l_{j-1}}{l_{j-1}} \quad \text{for } j = 1, \dots, n.$$

If the triangle $T_j(P)$ is bounded let l_j, s_j, p_j be the length of its sides (counterclockwise ordered) and $e_j = p_{j-1} + l_j + s_{j+1}$. We define L_n as the class of convex polygons in $\mathcal{P}_n$ satisfying the *outer ratio property*, i.e.

$$\frac{s_j\ e_{j-1}}{l_{j-1}(p_{j-2} + l_{j-1})} = \frac{p_j\ e_{j+1}}{l_{j+1}(s_{j+2} + l_{j+1})} \quad \text{for } j = 1, \dots, n.$$

The characterization of maximizers is then obtained by the following

Theorem 1. *Let $P^\star$ be a maximizers of F in $\mathcal{P}_n$ then $P^\star$ has both the inner equivalent triangle and diagonal-side property i.e $P^\star \in \Phi_n \cap I_n$.*

Theorem 2. *Let $P^\star$ be a maximizers of G in $\mathcal{P}_n$ for then $P^\star$ has both the outer equivalent triangle and outer ratio property i.e $P^\star \in \Gamma_n \cap L_n$.*

Theorem 3.

$$\Phi_n \cap I_n = \Gamma_n \cap L_n = \{\ \textit{affinely regular polygons}\ \}\ .$$

Extensions of F and G to the class of plane convex bodies are considered and again their maximizers are the affinely regular polygons.

Monika LUDWIG *(Technische Univ. Wien, Austria)*

A Classification of SL(n) invariant Valuations

Joint work with M. Reitzner

A classification of upper semicontinuous and SL(n) invariant valuations on the space of n-dimensional convex bodies is established. A consequence is a characterization of L_p-affine surface area.

Endre MAKAI, Jr. *(A. Rényi Inst. Math., Hungary)*

Maximal k-sections of convex bodies in $\mathbb{R}^n$

Joint work with V. L. Dol'nikov, R. J. Gardner, H. Martini, J. Matoušek, S. Vrećica, R. Živaljević

We consider $\mathbb{R}^n$, where we always suppose $n \geq 2$.

Definition 1. *Let $K \subset \mathbb{R}^n$ be a convex body (i.e., compact, convex, with non-empty interior). Let $L_k \subset \mathbb{R}^n$ be an affine k-subspace, where $1 \leq k \leq n-1$. Then the set $K \cap L_k$ is called a maximal k-section of K if $\forall x \in \mathbb{R}^n \;\; V_k(K \cap L_k) \geq V_k(K \cap (L_k + x))$. ($V_k$ means k-dimensional volume.)*

Example 1. *If K is a ball, and $L_k \subset \mathbb{R}^n$ is an affine k-subspace, then $K \cap L_k$ is a maximal k-section of K iff L_k contains the centre of K.*

For $k = 1$, $K \cap L_1$ is a maximal 1-section of K iff it is an *affine diameter of* K, i.e., iff its two endpoints are contained in two distinct parallel supporting hyperplanes of K.

Theorem 1 (P. C. Hammer, C. M. Petty, J. M. Crotty)**.** *Let $K \subset \mathbb{R}^n$ be a convex body, and $x \in \mathbb{R}^n$. Then there exists a maximal 1-section $K \cap L_1$ of K, such that $L_1 \ni x$.*

Theorem 2. *Let $K_1, K_2 \subset \mathbb{R}^n$ be convex bodies. Then there exists an affine 1-subspace $L_1 \subset \mathbb{R}^n$, such that $K_i \cap L_1$ is a maximal 1-section of K_i, $i = 1, 2$.*

Theorem 2 implies Theorem 1 (choose $K_1 = K$, K_2 a ball of centre x).

Theorem 3. *Let $K \subset \mathbb{R}^n$ be a convex body and $x \in \mathbb{R}^n$. Then there exists a maximal $(n-1)$-section $K \cap L_{n-1}$ of K, such that $L_{n-1} \ni x$. Moreover, the set of (both) unit normals of such affine $(n-1)$-subspaces L_{n-1} is not contained in any compact C^1 $(n-2)$-submanifold of S^{n-1} with $(n-2)$-volume less than that of S^{n-2}. The estimate is sharp.*

Theorem 4. *Let $K \subset \mathbb{R}^n$ be a convex body, $x \in \mathbb{R}^n$ and $1 < k < n-1$ an integer. Then there exists a maximal k-section $K \cap L_k$ of K, such that $L_k \ni x$. Moreover, the set of translates of such affine k-subspaces, containing 0, is not contained in any compact C^1 $(k-1)(n-k)$-submanifold of the Grassmannian $Gr_{n,k}$ of all linear k-subspaces of $\mathbb{R}^n$, with $(k-1)(n-k)$-volume (w.r.t. a natural $O(n)$-invariant Riemannian metric on $Gr_{n.k}$) less than some positive constant $c_{n,k}$. The value of the dimension of the submanifold is sharp.*

Theorem 5. *Let $K_1, ..., K_{k+1} \subset \mathbb{R}^n$ be convex bodies, where $1 \leq k \leq n-1$ is an integer. Then there exists an affine k-subspace $L_k \subset \mathbb{R}^n$, such that $K_i \cap L_k$ is a maximal k-section of K_i, $i = 1, ..., k+1$.*

For $k = 1$ Theorem 5 reduces to Theorem 2.

Theorem 6. *Let $K_1, ..., K_{l+1} \subset \mathbb{R}^n$ be convex bodies, where $1 \leq k \leq n-1$ and $1 \leq l \leq k+1$ are integers. Then the set of affine k-subspaces $L_k \subset \mathbb{R}^n$, such that $K_i \cap L_k$ is a maximal k-section of K_i, $i = 1, ..., l+1$, cannot be included into a compact C^1 $(k+1-l)(n-k)$-submanifold of the manifold of all affine k-subspaces of $\mathbb{R}^n$, of $(k+1-l)(n-k)$-volume less than some positive constant $c_{n,k,l}$. (Here we identify the affine k-subspaces of $\mathbb{R}^n$ with linear $(k+1)$-subspaces of $\mathbb{R}^{n+1}$, via $L_k \to lin\{(x_1, ..., x_n, 1) \mid (x_1, ..., x_n) \in L_k\}$, where lin is linear hull. Moreover, volume is meant w.r.t. the natural $O(n+1)$-invariant Riemannian metric on $Gr_{n+1,k+1}$, analogous to that of Theorem 4 for $Gr_{n,k}$.) The value of the dimension of the submanifold is sharp.*

For $l = k+1$ Theorem 6 reduces to Theorem 5.

Remark 1. *For several of our statements we have counterexamples if convexity is not assumed.*

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Cinzia MIORI *(Univ. de Alicante, Spain)*

Chords halving the area of a planar convex set

Joint work with A. Grüne, E. Martínez, S. Segura Gomis

Ebbers-Baumann, Grüne and Klein have studied the chords halving the perimeter of a planar bounded convex set. The aim of this communication is to study some inequalities describing geometric properties of the chords halving the area of such sets. In particular we obtain upper and lower bounds of the ratio between either the maximal or the minimal distance halving area with the six classical geometric magnitudes. In many cases we determine the extremal sets which attain the bounds or at least give some examples of these sets; in other cases we obtain bounds that are not tight. We also state some conjectures.

Luis MONTEJANO *(Univ. Nacional Autónoma de México, México)*

Shaken False Centre Theorems

Let K be a convex body and let P be a point. Suppose that every section of K through P is centrally symmetric, then Rogers proved that K is centrally symmetric, although P may not be the centre of K. If this is the case, Aitchison, Petty and Rogers, and Larman proved that K must be an ellipsoid. Suppose now that for every direction we can choose continuously a section of K that is centrally symmetric, if K is strictly convex, then Montejano proved that K must by centrally symmetric. Consider now the following example: Let D be a solid sphere centered at the origin in which two symmetric caps are deleted. Then, D is centrally symmetric with respect the origin and has a lot of circular sections whose center is not the origin. In fact, we can choose continuously, for every direction, a section of D which is centrally symmetric in such a way that not all these sections pass through the origin. Nevertheless, no matter how we choose these sections, there is always many of them that necessarily pass through the origin. For those sections, of course, we have not impose really any condition which explain the fact that D is not a quadric elsewhere.

Let K be a convex body centrally symmetric. Suppose that for every direction we can choose continuously a section of K which is centrally symmetric or suppose, alternatively, that there is a convex set M contained in the interior of K with the property that every tangent plane of M intersects K in a centrally symmetric section. The purpose of this talk is to discuss under which additional conditions we are able to conclude that K is an ellipsoid.

Vladimir I. OLIKER *(Univ. of Atlanta, U.S.A)*

Variational solutions of some problems in convexity via Monge-Kantorovich optimal mass transport theory

In his classical book on convex polyhedra [1] A.D. Aleksandrov raises a general question of finding variational formulations and solutions to problems of existence and uniqueness of convex hypersurfaces in Euclidean space with prescribed geometric data, such as integral Gauss curvature, area, etc. An example of such problem and its variational solution is the famous Minkowski problem.

In this talk I plan to show that there is a variational principle that can be applied in an almost canonical way to several such problems to prove existence and uniqueness. In particular, the Aleksandrov problem of existence of complete noncompact and compact convex hypersurfaces with prescribed integral Gauss curvature can be solved by this method. The construction of the required functional is motivated by the Monge-Kantorovich optimal mass transport theory. If time permits, I will also discuss some problems in geometrical optics for which methods from convexity theory combined with the above variational approach lead to existence and uniqueness results and often to efficient way for calculating solutions numerically.

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Elena OURNYCHEVA *(Hebrew Univ., Israel)*

The Composite Cosine Transform on the Stiefel Manifold

Joint work with B. Rubin

The λ-cosine transform on the unit sphere S^{n-1} in $\mathbb{R}^n$ is defined by

$$(T^\lambda f)(u) = \int\limits_{S^{n-1}} f(v)|v \cdot u|^\lambda \, dv, \qquad u \in S^{n-1}.$$

It has numerous applications to convex geometry, probability and Banach space theory. We introduce a new integral transform $T^{\boldsymbol{\lambda}} f, \boldsymbol{\lambda} \in \mathbb{C}^m$, which generalizes the previous one for functions on the Stiefel and Grassmann manifolds. We call it the composite cosine transform, by taking into account that its kernel agrees with the composite power function of the cone of positive definite symmetric matrices. Our aim is to describe the set of all $\boldsymbol{\lambda} \in \mathbb{C}^m$ for which $T^{\boldsymbol{\lambda}}$ is injective on the space of integrable functions. We obtain the precise description of this set in some important cases, in particular, for λ-cosine transforms on Grassmann manifolds. The main tools are the classical Fourier analysis of functions of matrix argument and the relevant zeta integrals. This approach differs from those of other authors and gives more complete results. In particular, we give an alternative proof of the result due to P. Goodey and R. Howard on non-injectivity of Matheron's transform (the case $\lambda = 1$) on the Grassmann manifold.

Shlomo REISNER *(Univ. of Haifa, Israel)*

On certain variations of convex bodies

Joint work with M. Meyer

A *shadow system* along a direction $v \in \mathbb{S}^{n-1}$ (introduced by Rogers and Shephard) is a family of convex sets $K_t \subset \mathbb{R}^n$ which is defined by $K_t = \text{conv}\{x + \alpha(x)tv\,;\, x \in A \subset \mathbb{R}^n,\, t \in [a,b]\}$, where A is a bounded set, α is a bounded function on A and $[a,b]$ is an interval. S. Campi and P. Gronchi proved that, for K_t as above, if for all $t \in [a,b]$ K_t is an origin symmetric convex body in $\mathbb{R}^n$, then $\text{vol}(K_t^*)^{-1}$ is a convex function of t where K_t^* is the polar (about the origin) body of K_t. We prove this result without the symmetry assumption, provided that K_t^* is understood as the polar body of K_t about its Santaló point. We also characterize, subject to certain assumptions, the case when $\text{vol}(K_t^*)$ is constant. We apply this to prove (exact) reverse Santaló inequality for polytopes in $\mathbb{R}^n$ with few vertices.

Boris RUBIN *(Louisiana State Univ., U.S.A and Hebrew Univ., Israel)*

On the generalized Busemann-Petty problem for k-dimensional sections of convex bodies in $\mathbb{R}^n$

The generalized Busemann-Petty problem asks whether origin-symmetric convex bodies in $\mathbb{R}^n$ with smaller k-dimensional sections necessarily have smaller volume. The Fourier transform approach to this problem is due to A. Koldobsky. I will be speaking about alternative approach via analytic families of intertwining operators. This enables us to give new simple proofs of some known results and interpret the latter from different point of view.

Paolo SALANI *(Univ. di Firenze, Italy)*

Convexity properties of solutions to nonlinear elliptic equations

Joint work with Paola Cuoghi

We find suitable assumptions which assure that the quasi-concave envelope u^* of a solution (or a subsolution) u of an elliptic equation $F(x, u, \nabla u, D^2u) = 0$ (possibly fully nonlinear) is a viscosity subsolution of the same equation. We apply this result to study the convexity of level sets of solutions to elliptic Dirichlet problems in a convex ring $\Omega = \Omega_0 \setminus \overline{\Omega}_1$.

Franz SCHUSTER *(Technische Univ. Wien, Austria)*

Characterization of rotation equivariant additive mappings

In this talk we discuss continuous mappings from the space of convex bodies in Euclidean n-dimensional space into itself that are additive with respect to Minkowski and Blaschke addition and commute with rotations. The unifying feature of such mappings is a representation as a convolution operator and as a consequence a multiplier property. The pertinent representations concerning Minkowski and Blaschke endomorphisms will be reviewed and a corresponding theorem on Blaschke Minkowski additive mappings will be presented. Finally several properties will be derived from the established representation theorem.

Alina STANCU *(Polytechnic Univ. of New York, U.S.A)*

On Hadwiger's containment formula

Hadwiger's containment theorem states that if K and L are convex bodies in $\mathbb{R}^2$ with non-empty interiors such that

$$\Delta(K, L) := 2\pi(\text{Area}(K) + \text{Area}(L)) - \text{Length}(\partial K) \cdot \text{Length}(\partial L) > 0,$$

then there exists a Euclidean motion g such that either $K \subseteq \text{int}\, gL$ or $L \subseteq \text{int}\, gK$.

We investigate the existence of stronger Hadwiger-type inequalities $\Delta(K, L) > f(K, L)$, with $f : \mathcal{K}^2 \times \mathcal{K}^2 \to \mathbb{R}_-$, which imply $K \subseteq \text{int}\, gL$ (or $L \subseteq \text{int}\, gK$).

Konrad J. SWANEPOEL *(Univ. of South Africa, South Africa)*

An illumination problem of K. Bezdek

We consider an illumination problem introduced by Károly Bezdek [1]. Let K be a d-dimensional convex body. A point $p \notin K$ *illuminates* a point q on the boundary of K if the ray $\{\lambda p + (1-\lambda)q : \lambda < 0\}$ intersects the interior of K. A set of points $P \subseteq \mathbb{R}^d \setminus K$ *illuminates* K if each boundary point of K is illuminated by some point in P. If $K = -K$ then K defines a norm $\|\cdot\|_K$. Define

$$b(K) := \inf \left\{ \sum_i \|p_i\|_K : \{p_i\} \text{ illuminates } K \right\}.$$

Bezdek asked for an upper bound for $b(K)$ depending only on d.

Theorem 1. *For any centred d-dimensional convex body K we have*

$$b(K) < 2^{d+1}e(d+1)d(\log d + \log\log d + 5).$$

Note that if K is a d-cube then $b(K) = 2^d$. The proof uses efficient coverings of d-space by convex bodies due to Rogers.

Our second result concerns Steiner minimal trees. Let $v(K)$ be the maximum degree of a vertex, and $s(K)$ of a Steiner point, in a Steiner minimal tree in a d-dimensional normed space with unit ball K. Morgan [3] conjectured that $s(K) \leq 2^d$ (attained if K is a d-cube), and Cieslik [2] conjectured $v(K) \leq 2(2^d - 1)$.

Theorem 2. *For any centred d-dimensional convex body K we have* $s(K) \leq v(K) \leq b(K)$.

Combined with the above estimate of $b(K)$, this improves the previously best known upper bound $v(K) < 3^d$.

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Istvan TALATA *(Ybl College of St. István Univ., Hungary)*

On m-Sparse subsets of the unit cube in l_p-spaces

Let $\mathcal{M}^d = (\mathbb{R}^d, \|.\|)$ be a d-dimensional normed space, $d \geq 2$. Consider an arbitrary convex body K in $\mathcal{M}^d$. For any integer $m \geq 1$, we say that a multiset S is an m-sparse subset of K if $S \subseteq K$ and no $m+1$ points of S can be covered simultaneously by the interior of a translate of the unit ball $\{x \in \mathbb{R}^d \mid \|x\| \leq 1\}$ of $\mathcal{M}^d$. Let $C_d = [-1, 1]^d$ be a cube of edge length 2, whose edges are parallel to the coordinate axes. Note that C_d is the unit ball with respect to the d-dimensional l_∞-norm.

For any fixed integer $m \geq 1$, let S be an arbitrary m-sparse subset of C_d in the d-dimensional space equipped with the l_p-norm. It is easy to show for the cardinality of S that $|S| \leq m2^d$ when $p > \log_2 d$, and the upper bound is sharp. The main result presented in the talk is the analogous inequality for $p = \log_2 d$: We have $|S| \leq m(2^d + 1)$ when $p = \log_2 d$, and the upper bound is sharp. Moreover, we prove that the extremal configuration in this case is unique. We also discuss the stability of this inequality.

Cristina TROMBETTI *(Univ. di Napoli, Italy)*

A quantitative version of the isoperimetric inequality: the anisotropic case

We prove a stability result for the isoperimetric problem in the anisotropic case. More precisely given a set E of finite perimeter, defined the "anisotropic" isoperimetric deficit $\Delta(E)$, we prove that if $\Delta(E)$ is small enough, then the set E is close, in measure, to the Wulff shape set.

Christophe WEIBEL *(École Polytechnique Fédérale de Lausanne, Swiss)*

Minkowski Sums of Perfectly Centered Polytopes

Joint work with K. Fukuda

Minkowski sums have applications in many domains, ranging from mechanics to algebra. It is in general difficult to estimate the number of faces of the result, even if we know the face lattices of the summands. The objective of this paper is to present a special class of polytopes called **perfectly centered** and the combinatorial properties of the sum with their own dual. In particular, we have an exact formula for counting k-faces in terms of the face lattice of a given perfectly centered polytope. In fact the face lattice itself of the sum is easily deducible. Successive sums of a polytope with its own dual has been shown by Yuri Nesterov to converge towards a sphere, which could be useful for optimization purposes.

Vladyslav YASKIN *(Univ. of Missouri, U.S.A)*

A solution to the lower dimensional Busemann-Petty problem in the hyperbolic space

The lower dimensional Busemann-Petty problem asks whether origin symmetric convex bodies in $\mathbb{R}^n$ with smaller volume of all k-dimensional sections necessarily have smaller volume. As proved by Bourgain and Zhang, the answer to this question is negative if $k > 3$. The problem is still open for $k = 2, 3$.

We formulate and completely solve the lower dimensional Busemann-Petty problem in the hyperbolic space $\mathbb{H}^n$.

Maryna YASKINA *(Univ. of Missouri, U.S.A)*

The geometry of L_0

Joint work with N. J. Kalton, A. Koldobsky, V.Yaskin

Suppose that we have the unit Euclidean ball in $\mathcal{R}^n$ and construct new bodies using three operations - linear transformations, closure in the radial metric and multiplicative summation defined by $\|x\|_{K+_0L} = \sqrt{\|x\|_K\|x\|_L}$. We prove that in dimension 3 this procedure gives all origin symmetric convex bodies, while this is no longer true in dimensions 4 and higher. We introduce the concept of embedding of a normed space in L_0 that naturally extends the corresponding properties of L_p-spaces with $p \neq 0$, and show that the procedure described above gives exactly the unit balls of subspaces of L_0 in every dimension. We provide Fourier analytic and geometric characterizations of spaces embedding in L_0, and prove several facts confirming the place of L_0 in the scale of L_p-spaces.

Tudor ZAMFIRESCU *(Univ. Dortmund, Germany)*

Inequalities involving the measure of critical sets

Several things are known about the measure and dimension of the cut loci on Alexandrov surfaces. Their length can easily be infinite. The critical sets, i.e. the sets of critical points associated to arbitrary points of the surface, are subsets of the respective cut loci. Their measure must verify much stricter inequalities. The convex case is especially investigated. For the general case, inequalities involving the total positive Gauss curvature are presented.

Gaoyong ZHANG *(Polytechnic Univ. of New York, U.S.A)*

On a volume inequality

Joint work with E. Lutwak and D. Yang

A new ellipsoid associated with convex bodies was introduced by Lutwak, Yang and Zhang, which is dual to the classical Legendre ellipsoid. They proved a volume inequality of the ellipsoid by using Ball's volume ratio inequality of the John ellipsoid. A new direct proof of the volume inequality will be given. Applications to the reverse isoperimetric inequality and Schneider's projection problem will be discussed.

Jiazu ZHOU *(Guizhou Normal Univ., China)*

On the Willmore functional of submanifolds

The well known Willmore theorem states:

Let M be a closed surface in $\mathbf{R}^3$ and H be the mean curvature of M. Denote by dA the volume element of M, then

$$\int_M H^2 dA \geq 4\pi,$$

with equality if and only if M is a standard sphere.

It has not been known for the low bound of the Willmore functional $\int_M H^2 dA$ for the submanifold M of $dim(M) \neq 2$ since last century. Recent applications of kinematic formulas in integral geometry inspires some results for submanifold M of $dim(M) \neq 2$. The results could be some Bonnesen-type geometric inequalities of M, that is, inequalities involving the volume, curvature integrals, radius of the inscribed ball and the circumscribed ball of M.

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