

One-parameter families of spatial PH quintic interpolants, of identical arc length, defined by keeping  $\phi_2 - \phi_0$  constant, and varying only  $\frac{1}{2}(\phi_0 + \phi_2)$ 

rational rotation-minimizing frame (RRMF) curves

rational frames  $(\mathbf{t}, \mathbf{u}, \mathbf{v})$  with angular velocity satisfying  $\boldsymbol{\omega} \cdot \mathbf{t} \equiv 0$ 



Frenet





Frenet frame (center) & rotation-minimizing frame (right) on space curve



motion of an ellipsoid oriented by Frenet & rotation-minimizing frames



sudden reversal of Frenet frame through an inflection point



surface constructed by sweeping an ellipse along a space curve using Frenet frame (center) & rotation-minimizing frame (right)

## Hermite interpolation by quintic RRMF curves

given initial, final positions & frames  $\mathbf{p}_i$  &  $(\mathbf{t}_i, \mathbf{u}_i, \mathbf{v}_i)$  and  $\mathbf{p}_f$  &  $(\mathbf{t}_f, \mathbf{u}_f, \mathbf{v}_f)$ compute RRMF quintic  $\mathbf{r}(\xi)$  & frame  $(\mathbf{t}(\xi), \mathbf{u}(\xi), \mathbf{v}(\xi))$  interpolating data



two distinct rational rotation-minimizing motions interpolating given data

## Hermite interpolation by degree 7 RRMF curves

given initial, final positions & frames  $\mathbf{p}_i$  &  $(\mathbf{t}_i, \mathbf{u}_i, \mathbf{v}_i)$  and  $\mathbf{p}_f$  &  $(\mathbf{t}_f, \mathbf{u}_f, \mathbf{v}_f)$ find degree 7 RRMF curve  $\mathbf{r}(\xi)$  & frame  $(\mathbf{t}(\xi), \mathbf{u}(\xi), \mathbf{v}(\xi))$  matching data



two distinct degree 7 PH curves with ERF = RMF interpolating given data

## further example with degree 7 RRMF curves



two distinct degree 7 PH curves with ERF = RMF interpolating given data

shapes optimized with respect to two residual free parameters  $w_i, w_f$