Fischer decompositions in Hermitean Clifford analysis

F. Brackx

A result of E. Fischer states that, given a homogeneous polynomial $q(x)$, $x \in \mathbf{R}^m$, every homogeneous polynomial $P_k(x)$ of degree k can be uniquely decomposed as $P_k(x) = Q_k(x) + q(x)R(x)$, where $Q_k(x)$ is homogeneous of degree k, satisfying $q(D)Q_k = 0$, D being the differential operator corresponding to x through Fourier identification, and $R(x)$ is a homogeneous polynomial of suitable degree. If in particular $q(x) = ||x||^2$, then $q(D)$ is the Laplacian and Q_k is harmonic, leading to the well–known decomposition of the spaces of complex valued homogeneous polynomials into spaces of complex valued harmonic homogeneous polynomials.

Fischer decompositions of the spaces of Clifford algebra or spinor valued homogeneous polynomials into spaces of monogenic homogeneous polynomials are fundamental in Clifford analysis and very well known. In the more recent branch of Hermitean Clifford analysis, the rotational invariance has been broken by introducing a complex structure J on Euclidean space and a corresponding second Dirac operator $\underline{\partial}_J$, leading to the system of equations $\underline{\partial} f = 0 = \underline{\partial}_J f$ expressing so-called Hermitean monogenicity. The invariance of this system is reduced to the unitary group. In this talk a number of Fischer decompositions into $\mathrm{U}(n)$-irrucibles involving homogeneous Hermitean monogenic polynomials are discussed.

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