# On Riesz and Moisil-Théodoresco systems

by

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Abstract:

**Let $\mathbb{R}^{0,m+1}$ be the space $\mathbb{R}^{m+1}$ provided with a quadratic form of signature $(0, m+1)$ and let $\mathbb{R}_{0,m+1}$ be the universal Clifford algebra constructed over $\mathbb{R}^{0,m+1}$. Furthermore, let for $0 \leq r \leq m+1$ fixed, $\mathbb{R}_{0,m+1}^{(r)}$ be the space of $r$-vectors in $\mathbb{R}_{0,m+1}$ and let for $p, q \in \mathbb{N}$ with $p < q$ and $r + 2q \leq m+1$, $\mathbb{R}_{0,m+1}^{(r,p,q)}$ be the subspace $\mathbb{R}_{0,m+1}^{(r,p,q)} = \sum_{j=p}^{q} \oplus \mathbb{R}_{0,m+1}^{(r+2j)}$ of $\mathbb{R}_{0,m+1}$.**

**If $\Omega \subset \mathbb{R}^{m+1}$ is open, then the following spaces of nullsolutions to the Dirac operator $\partial_x$ in $\mathbb{R}^{m+1}$ are considered.**

**(I)** **The space $M(\Omega; \mathbb{R}_{0,m+1}^{(r)})$ $(0 < r < m+1)$, i.e. the space of $\mathbb{R}_{0,m+1}^{(r)}$-valued smooth functions $F_r$ in $\Omega$ satisfying the equation**

$$\partial_x F_r = 0 \tag{1}$$

**The system corresponding to (1) generalizes the classical Riesz system in $\mathbb{R}^3$.**

**(II)** **The space $M(\Omega; \mathbb{R}_{0,m+1}^{(r,p,q)})$, i.e. the space of $\mathbb{R}_{0,m+1}^{(r,p,q)}$-valued smooth functions $W$ in $\Omega$ satisfying the equation**

$$\partial_x W = 0 \tag{2}$$

**The system corresponding to (2) generalizes the classical Moisil-Théodoresco system in $\mathbb{R}^3$.**

**In this talk, the following problems are dealt with:**

**(P1)** **The construction of harmonic potentials to solutions of the equations (1) and (2).**

**(P2)** **The characterization of boundary values of elements belonging to the Hardy spaces $H^2(\mathbb{R}_+^{m+1}; \mathbb{R}_{0,m+1}^{(r)})$ and $H^2(\mathbb{R}_+^{m+1}; \mathbb{R}_{0,m+1}^{(r,p,q)})$, $\mathbb{R}_+^{m+1}$ being the upper half-space in $\mathbb{R}^{m+1}$.**