On theory of actions applied to hyperholomorphic Bergman spaces Abstract

The present work gives an original interpretation, using theory of actions of groups on sets, of some properties of the weighted hyperholomorphic Bergman spaces which were recently presented in [2].

The main idea is to work with actions of subgroups of the group of the Möbius transformations (\mathbb{R}^2 , \mathbb{R}^4) on sets of Bergman spaces (holomorphic, hyperholomorphic), and to present the orbite and the stabilizer of some elements.

The complex case is the following:

By Γ we mean the group of biholomorphic functions which preserve the unit disk \mathbb{B}^2 .

1 Proposition. The function $\delta : \Gamma \times \mathcal{A}^2(\mathbb{B}^2) \to \mathcal{A}^2(\mathbb{B}^2)$ given by

$$\delta(\alpha, f) = \alpha \cdot f := {}^{\alpha'} M \circ W_{\alpha}[f] = \alpha' f \circ \alpha, \quad \forall f \in \mathcal{A}^2(\mathbb{B}^2), \quad \forall \alpha \in \Gamma$$

is an action of the group $(\Gamma, *)$ on the $\mathcal{A}^2(\mathbb{B}^2)$.

2 Examples.

- 1. For the function identically equal to zero, there holds that $O_0 = \{0\}$ and $E_0 = \Gamma$.
- 2. The orbit and the stabilizer of a constant function $c \in \mathbb{C}$ are, respectively, $O_c = \{ c\alpha' \mid \alpha \in \Gamma \}$ and $E_c = \{ \alpha \in \Gamma \mid c\alpha' = c \} = \{ I \}.$

3. If
$$f(z) = z$$
, $\forall z \in \mathbb{B}^2$, then $O_f = \{ \alpha' \alpha \mid \alpha \in \Gamma \}$ and $E_f = \{ \pm I \}$

4. Let $w \in \mathbb{B}^2$,

$$O_{\mathcal{B}_{\mathbb{B}^2}(\cdot,w)} = \{ \mathcal{B}_{\mathbb{B}^2}(\cdot,\alpha^{-1}(w))(\overline{\alpha'(\alpha^{-1}(w))})^{-1} \mid \alpha \in \Gamma \},$$

and

$$E_{\mathcal{B}_{\mathbb{R}^2}(\cdot,w)} = \{ \alpha \in \Gamma \mid \alpha(w) = w \text{ and } \alpha'(w) = 1 \}.$$

References

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