The Cauchy–Kovalevskaya extension theorem in Clifford analysis

H. De Schepper

The Cauchy–Kovalevskaya extension theorem is well–known: the classical idea behind it is to characterize solutions of suitable (systems of) PDE's by their restriction, sometimes together with the restrictions of some of their derivatives, to a submanifold of codimension one. In particular, it follows from this theorem that a holomorphic function in the complex plane is completely determined by its restriction to the real axis. This holomorphic CK–extension principle has been elegantly generalized to higher dimension in the framework of Clifford analysis, a higher dimensional function theory centered around the notion of a monogenic function, i.e. a function defined on $\mathbf{R}^m$ and taking values in the real Clifford algebra $\mathbf{R}_{0,m}$, which moreover is a null solution of the well–known Dirac operator. Completely similar to the holomorphic case, a monogenic function in an appropriate region of $\mathbf{R}^m$ will be completely determined by its restriction to the hyperplane $x_m = 0$.

More recently Hermitean Clifford analysis has emerged as a new branch of Clifford analysis, which focusses on the simultaneous null solutions, called Hermitean monogenic functions, of two Hermitean conjugate complex Dirac operators. The functions considered now are defined on $\mathbf{C}^n$ and take their values in the complex Clifford algebra $\mathbf{C}_{2n}$. We establish a CK–extension theorem for Hermitean monogenic polynomials. The minimal number of initial polynomials in $\mathbf{C}^{n-1}$, needed in order to obtain a unique Hermitean monogenic extension to $\mathbf{C}^n$, is determined, along with the compatibility conditions they have to satisfy. The corresponding Hermitean monogenic polynomial is explicitly constructed. Moreover, as the CK–extension map is an isomorphism between the spaces of initial polynomials subject to compatibility conditions and the corresponding spaces of Hermitean monogenics, we will use this result for a dimensional analysis.

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