More about quaternionic hyperderivatives

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In one complex variable theory, the derivative of a holomorphic function is equal to the directional derivative along any direction. The deep analog of this fact in quaternionic analysis consists of coincidence of the quaternionic hyperderivative and of the three-dimensional directional hyperderivatives. The compatibility in both cases is due to the fact that the directions considered have the corresponding hyperdimensions: one in the complex case and three in the quaternionic one. In $\mathbb{R}^4$ there are also two-dimensional planes and one-dimensional lines, so one is led to ask about the hyperderivatives along these directions in such a way that they would be again consistent with hyperderivability of hyperholomorphic functions. It turns out that it is not a trivial task to introduce an adequate notion (see an attempt in [1] and [2]). The matter is that one- and two-dimensional sets in $\mathbb{R}^4$ are "too thin"; in particular one may lose a "good compatibility" with the quaternionic hyperderivative. In this talk there will be presented a refinement of the notion of two-dimensional directional hyperderivative, which is compatible with the quaternionic hyperderivative in the same meaning as above.

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References.

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