The class of Clifford-Fourier transforms

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The classical Fourier transform is a mathematical tool of the utmost importance in harmonic analysis and has of course an enormous number of applications in virtually all branches of physics and engineering. On the other hand, the theory of Clifford analysis, in its most basic form, is a refinement of the theory of harmonic analysis in m-dimensional Euclidean space. By introducing the so-called Dirac operator, the square of which equals the Laplace operator, one introduces the the notion of monogenic functions. These are at the same time, a refinement of harmonic functions and a generalization of holomorphic functions in one complex variable.

In [1] a genuine multi-dimensional Fourier transform within the context of Clifford analysis, the so-called Clifford-Fourier transform, was constructed. It is given in terms of an operator exponential, or alternatively, by a series representation. For the even dimensional case, it was proved recently (see [3]) that the kernel of this integral transform takes the form of a finite sum of Bessel functions. In the case of odd dimension, it is shown that it is sufficient to identify the kernel in dimension three, from which kernels in higher odd dimensions can be deduced by taking suitable derivatives.

Similar to the classical case, the kernel of the Clifford-Fourier transform satisfies a system of differential equations, which we call "the Clifford-Fourier system". In this talk, we will determine general parabivector-valued solutions of this system, thus obtaining a whole class of Clifford-Fourier transforms. Naturally, the original Clifford-Fourier kernel is reobtained, but also the Fourier-Bessel kernel (see [2]) belongs to this class. The latter kernel, which in the two-dimensional case coincides with the Clifford-Fourier kernel of [1], is obtained by leaving an exponential factor out of the so-called Bessel-exponential, introduced by Sommen who recently used it to introduce Clifford generalizations of the classical Fourier-Borel transform.

Moreover, by expressing the newly obtained solutions of the Clifford-Fourier system as derivatives of the Fourier-Bessel kernel, we are able to determine the eigenvalues of an L_2-basis consisting of generalized Clifford-Hermite functions under the action of the new Clifford-Fourier transforms.

References

[1] F. Brackx, N. De Schepper and F. Sommen, The Clifford-Fourier transform. *J. Fourier Anal. Appl.* **11** (2005), 669–681.

[2] F. Brackx, N. De Schepper and F. Sommen, The Fourier-Bessel transform. In: (digital) Proceedings 18th International Conference on the Applications of Computer Science and Mathematics in Architecture and Civil Engineering, K. Gürlebeck and C. Könke (eds.), 7-9 July 2009, Bauhaus-Universität Weimar.

[3] H. De Bie and Y. Xu, On the Clifford-Fourier transform. arXiv:1003.0689, 30 pages.