The Fueter mapping theorem in integral form and its inverse

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1 Abstract

The Fueter mapping theorem is a classical result in quaternionic and Clifford analysis which has been investigated by several authors, at various degrees of generality. In recent times we have proved an integral representation formula for this theorem. Specifically, given a function f of the form $f = \alpha + \underline{\omega}\beta$ (where α, β satisfy the Cauchy-Riemann equations) we represent in integral form the axially monogenic function $\breve{f} = A + \underline{\omega}B$ (where A, B satisfy the Vekua's system) such that $\breve{f}(x) = \Delta^{\frac{n-1}{2}} f(x)$ where Δ is the Laplace operator in dimension $n+1$.

We have also solved the inverse problem: given an axially monogenic function $\breve{f}$ determine a function of the form $f = \alpha + \underline{\omega}\beta$ (called Fueter's primitive of $\breve{f}$) such that $\breve{f}(x) = \Delta^{\frac{n-1}{2}} f(x)$. We have proved an integral representation formula for f in terms of $\breve{f}$ which we call the inverse Fueter mapping theorem (in integral form). We also discuss some further generalizations of our approach.

This is a report on joint work with F. Colombo and F. Sommen.