Slice regular Laurent series and classification of singularities

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In [3], G. Gentili and D.C. Struppa started developing a theory of quaternion-valued functions of one quaternionic variable inspired by a work of Cullen. The class of functions considered, now called *slice regular* functions, includes power series of type $\sum_{n\in\mathbb{N}} q^n a_n$ on their balls of convergence $B(0,R) = \{q \in \mathbb{H} : |q| < R\}$. Conversely, every slice regular function on such a ball expands into a power series of the same type. If we change the center of the series, however, the behaviour is not quite as trivial, as shown in [2]. Indeed, the set of convergence is much more bizarre when the center of the series does not lie on the real axis. The construction of slice regular Laurent series and expansions, which we undertook in [4], leads to convergence properties that are even more extravagant. As one may imagine, Laurent expansions immediately apply to the classification of singularities as removable, essential or as poles. The analogy with the complex case is superficial, since the properties of poles and essential singularities are more articulate than one may expect. This is true, in particular, for the analog of the Casorati-Weierstrass theorem.

References

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