Booklet of titles and abstracts

New Trends on Calculus of Variations and PDEs

Firenze–Montecatini, June 12-15 2017

Lucio Boccardo – Università di Roma "La Sapienza"

A variational approach to parabolic equations with L^1 data (in the spirit of Lichnevsky-Temam-Marcellini)

We will present a new approach (with respect to the results of the past century by Gallouët-Dall'Aglio-Orsina-Boccardo) to the existence of solutions of parabolic equations with L^1 data, by means of T-minima (introduced by the speaker in the elliptic framework). The approach to the parabolic problem follows the outlines of the papers by Lichnevsky-Temam and Bögelein-Duzaar-Marcellini.

Verena Bögelein – Universität Salzburg

Existence of variational solutions in non-cylindrical domains

In this talk we establish the existence of evolutionary variational solutions in non-cylindrical domains. Our model problem is a gradient flow of the form

$$\partial_t u - \mathrm{div} Df(Du) = 0$$

in a non-cylindrical bounded domain $E \subset \mathbb{R}^n \times [0, T]$ with zero Dirichlet boundary data. For the integrand f we only assume that it is coercive and convex. We prove the existence of variational solutions. The only assumption on the domain E is that it has zero boundary measure. In some special situations, for instance in the case of non-decreasing domains, we can show uniqueness of variational solutions and the existence of the time derivative in L^2. This is joint work with Frank Duzaar (Erlangen), Christoph Scheven (Duisburg-Essen), and Thomas Singer (Erlangen).

Arrigo Cellina – Università di Milano-Bicocca

On the regularity of solutions to the problem of elasto-plasticity

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Andrea Cianchi – Università di Firenze

Second-order L^2-regularity in nonlinear elliptic problems

A second-order regularity principle is presented for solutions to a class of quasilinear elliptic equations in divergence form, including the p-Laplace equation, with merely square-integrable right-hand side. Our result amounts to the existence and square integrability of the weak derivatives of the nonlinear expression of the gradient under the divergence operator. This provides a nonlinear counterpart of the classical L^2-coercivity theory for linear problems. Both local and global estimates are established. The latter apply to solutions to either Dirichlet or Neumann boundary value problems. Minimal regularity on the boundary of the domain is required. If the domain is convex, no regularity of its boundary is needed at all. This is a joint work with V.Maz'ya.

Bernard Dacorogna – EPFL Lausanne

On the best constant in Gaffney inequality

We discuss the value of the best constant in Gaffney inequality namely

$$\|\nabla\omega\|_{L^2}^2 \leq C\left(\|d\omega\|_{L^2}^2 + \|\delta\omega\|_{L^2}^2 + \|\omega\|_{L^2}^2\right)$$

where ω is a $k-$form which satisfies either $\nu \wedge \omega = 0$ or $\nu \lrcorner\, \omega = 0$ on $\partial\Omega$. When $k = 1$, the inequality is equivalent to

$$\|\nabla\omega\|_{L^2}^2 \leq C\left(\|\operatorname{curl}\omega\|_{L^2}^2 + \|\operatorname{div}\omega\|_{L^2}^2 + \|\omega\|_{L^2}^2\right).$$

This is a joint work with G. CSATO and S. SIL.

Gianni Dal Maso – SISSA

Homogenisation and Gamma-convergence of free-discontinuity problems

We study the Gamma-convergence of sequences of free-discontinuity functionals depending on vector-valued functions u which can be discontinuous across hypersurfaces whose shape and location are not known a priori. The main novelty of our result is that we work under very general assumptions on the integrands which, in particular, are not required to be periodic in the space variable. Further, we consider the case of surface integrands which are not bounded from below by the amplitude of the jump of u. We obtain three main results: compactness with respect to Gamma-convergence, representation of the Gamma-limit in an integral form and identification of its integrands, and homogenisation formulas without periodicity assumptions. In particular, the classical case of periodic homogenisation follows as a by-product of our analysis. Moreover, our result will be useful to study the case of stochastic homogenisation.

Emmanuele DiBenedetto – Vanderbilt University

Remarks on Local Regularity of Solutions to Anisotropic p-Laplacian Equations

For local solutions to quasi-linear versions of the anisotropic p-Laplacian equation, we present new forms of local boundedness estimates and some Hölder estimates when all the p's defining the anisotropy are sufficiently mutually close.

Frank Duzaar – Universität Erlangen

A variational approach to the porous medium equation

In this talk we establish an existence theory for the porous medium equation

$$\partial_t u^m - \Delta u = 0,$$

and more generally, for doubly nonlinear evolution equations of the type

$$\partial_t b(u) - \operatorname{div}\big(Df(Du)\big) = 0,$$

where f is only assumed to be coercive and convex. Our approach is purely variational and inspired by minimizing movements. It is flexible enough to deal also with obstacle problems with low regularity of the obstacle or time dependent boundary data. This is joint work with Verena Bögelein (Salzburg), Paolo Marcellini (Florence), and Christoph Scheven (Duisburg-Essen).

Irene Fonseca – Carnegie Mellon University

Variational Models for Image Processing

The mathematical treatment of image processing is strongly hinged on variational methods, partial differential equations, and machine learning. The bilevel scheme combines the principles of machine learning to adapt the model to a given data, while variational methods provide model-based approaches which are mathematically rigorous, yield stable solutions and error estimates. The combination of both leads to the study of weighted Ambrosio-Tortorelli and Mumford-Shah variational models for image processing.

Bernd Kawohl – Universität Köln

Remarks on the p-Laplacian operator

The p-Laplacian operator $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is a frequently studied object for $p \in (1,\infty)$. In my lecture I study limits $p \to \infty$ and $p \to 1$ of problems involving this operator. The limiting problems lead sometimes to purely geometric questions. I survey also some eigenvalue problems and comment on the normalized or game-theoretic p-Laplacian, which is given by $\Delta_p^N u := \frac{1}{p}|\nabla u|^{2-p}\Delta_p u$ and which is a convex combination of Δ_1^N and Δ_∞^N.

Juha Kinnunen – Aalto University

Definitions of supersolutions to the porous medium equation

We discuss nonnegative supersolutions to the porous medium equation

$$u_t - \Delta(u^m) = 0,$$

in the slow diffusion case $m > 1$. It is possible to define the notion of a supersolution in several different ways and interesting questions arise whether the various approaches coincide. We consider weak supersolutions, very weak supersolutions and m-supercaloric functions defined as lower semicontinuous functions obeying comparison principle. Yet another class of supersolutions is viscosity supersolutions, defined via pointwise touching test functions. These classes coincide under suitable assumptions. Our approach is based on Sobolev space estimates, continuity properties, comparison principles and obstacle problems.

Perron's method to show existence of a solution is based on super- and subsolutions. The upper and lower Perron solutions are indeed weak solutions of the porous medium equation. A central question is to determine when the upper and lower solutions are the same function. If this happens, the boundary function is called resolutive. We extend Wiener's resolutivity theorem for the porous medium equation by showing that nonnegative continuous boundary functions are resolutive in cylindrical domains. No regularity assumptions on the base of the cylinder are needed. The upper and lower solutions may still take the wrong boundary values. However, the Perron solution attains the correct boundary values, if the boundary of the cylinder is sufficiently regular. These topics are based on collaboration with Pekka Lehtelä, Peter Lindqvist and Teemu Lukkari.

Jan Kristensen – University of Oxford

Uniqueness results for minimizers of quasiconvex integrals

It is well-known that uniqueness can fail for minimizers of strongly quasiconvex integrals. In this talk I discuss various types of additional conditions that ensure uniqueness of minimizers. The talk is based on joint work with Judith Campos Cordero.

Giovanni Leoni – Carnegie Mellon University

Some recent results in singular perturbation problems

In this talk we will present a survey of some recent result on singular perturbation models and their applications to the study of phase transitions problems and partial differential equations.

Francesco Maggi – ICTP

Compactness of critical points for elliptic isoperimetric problems

We prove that unions of Wulff shapes are the only accumulation points of sequences of sets whose anisotropic mean curvature converge to a constant. As applications we can characterize local minimizers of energies consisting of an elliptic surface tension plus a potential energy, and we can prove a weak version of Alexandrov's theorem for non-smooth and non-elliptic surface tension energies. This is a joint work with Matias Delgadino, Cornelia Mihaila and Robin Neumayer.

Juan J. Manfredi – University of Pittsburgh

Properties of solutions to semilinear evolution equations in the Heisenberg group

We study solutions to semilinear parabolic equations in the Heisenberg group. We show uniqueness of viscosity solutions with exponential growth at infinity and prove Lipschitz and horizontal convexity preserving properties under appropriate assumptions. Counterexamples show that in general some properties that are well known for semilinear and fully nonlinear parabolic equations in the Euclidean spaces do not hold in the Heisenberg group. Several key inequalities are obtained with the help of a computer algebra system.

This is joint work with Qing Liu (Fukuoka) and Xiaodan Zhou (Wurcester)

Keith Miller – UC Berkeley

The Parabolic Equation of Prescribed Mean Curvature - Theorems and Conjectures

We consider the parabolic equation $\frac{du}{dt} = A(u) + h$, where A is mean curvature. This is longstanding joint work with Paolo Marcellini. We review old results for $h(x)$. Here there is very satisfactory Theory in our June 1997, J Diff Eqns paper, "Elliptic vs Parabolic..." . We showed that when $h(x)$ is "too-big", the solution continues growing and $u(x,t)/t$ tends to a "parabolic growth function" $v(x)$ which can be very neatly characterized. In particular, we study the extremal cap region on which the solution rises fastest. The case of nonlinear $h(u)$ yields intriguing new behaviors when h has two attractive zeros such as $h(u) = 10(u(1-(\frac{u}{2})^2))$. There are no proofs, but Conjectures with asymptotic analysis and very convincing graphics in 1D and 2D. The solutions form positive and negative caps joined by vertical walls which do not move once formed.

Connor Mooney – Columbia University

Finite time blowup for parabolic systems in the plane

We will discuss an example of finite time blowup from smooth data for a linear, uniformly parabolic system in two dimensions.

Umberto Mosco – Worcester Polytechnic Institute

Synchronized discrete models of sand pile type

We consider a dissipative system of fast interacting particles on increasing short-range lattices. The system is described by nonlinear difference equations on spatial grids coupled with time-impulsive transport equations with actualization. We prove that a critical state is reached in a finite time. Our discrete model connects the automata models of physics on sand piles and self-organized-criticality to recent PDE mathematical work on these topics.

Patrizia Pucci – Università di Perugia

Schrödinger-Hardy systems involving two fractional operators: recent results and open questions

A great attention has been drawn to the study of fractional and nonlocal problems of elliptic type, since they arise in a quite natural way in many different applications. The talk is focused on recent results on existence of nontrivial nonnegative entire solutions of some nonlocal systems which exhibit an intrinsic lack of compactness, occuring from different reasons. The main features of the systems treated are however the presence of the Hardy terms and the fact that the nonlinearities do not necessarily satisfy the Ambrosetti-Rabinowitz condition. Moreover, we consider systems including even critical nonlinear terms, as treated very recently in literature. The results raise, and leave open, a number of other intriguing questions, which are briefly presented.

Angkana Rüland – University of Oxford

Microstructures in Shape-Memory Alloys – Rigidity and Flexibility

Shape-memory materials undergo a first order, diffusionless phase transformation, in which symmetry is lost. Mathematically, they are often modelled by non-convex, multi-well energies within the framework of the calculus of variations. Minimizers of these energies are often subject to a fascinating dichotomy: While solutions with high regularity are often quite rigid, solutions with low regularity are in many cases very flexible. I will discuss this in the context of the cubic-to-orthorhombic phase transformation, where this dichotomy already arises for the geometrically linearized theory of elasticity. Further, I will present first results which quantify this dichotomy. This is based on joint work with C. Zillinger and B. Zwicknagl.

Carlo Sbordone – Università di Napoli “Federico II”

The limit of bi–Sobolev mappings

Let $f_j, f : \Omega \subset \mathbb{R}^2 \to \Omega' \subset \mathbb{R}^2$ be Sobolev homeomorphisms and suppose $f_j \rightharpoonup f$ weakly in $W^{1,1}(\Omega, \mathbb{R}^2)$. Then we can only conclude that $f^{-1} \in BV(\Omega'; \mathbb{R}^2)$. If f_j is bi–Sobolev and f_j^{-1} stay bounded in $W^{1,2}$ then f is bi–Sobolev and $f^{-1} \in W^{1,2}(\Omega', \mathbb{R}^2)$.

Vincenzo Vespri – Università di Firenze

Pointwise estimates for a class of degenerate/singular parabolic equations

We consider the Cauchy problem associated to a class of nonlinear degenerate/singular parabolic equations, whose prototype is the parabolic p-Laplacian ($\frac{2N}{N+1} < p < \infty$). In his seminal paper, after stating the Harnack estimates, Moser proved almost optimal estimates for the parabolic kernel by using the so called Harnack chain method. In the linear case sharp estimates come by using Nashs approach. Fabes and Stroock proved that Gaussian estimates are equivalent to a parabolic Harnack inequality. In several papers in collaboration with Bögelein, Calahorrano, Piro-Vernier and Ragnedda, by using the DiBenedetto-De Giorgi approach we prove optimal kernel estimates for quasilinear parabolic equations. Lastly we use these results to prove existence and sharp pointwise estimates from above and from below for the fundamental solutions.
