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Hamilton cycles and paths in vertex-transitive graphs—Current directions

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ABSTRACT

In this article current directions in solving Lovász's problem about the existence of Hamilton cycles and paths in connected vertex-transitive graphs are given.

1. Historical motivation

In 1969, Lovász [59] asked whether every finite connected vertex-transitive graph has a Hamilton path, that is, a simple path going through all vertices, thus tying together two seemingly unrelated concepts: traversability and symmetry of graphs. Arguably, however, the general problem of finding Hamilton paths and cycles in highly symmetric graphs may be much older, as it can be traced back to bell ringing, Gray codes and the knight's tour of a chessboard (see [1,21,38,48]). Lovász problem is, somewhat misleadingly, usually referred to as the Lovász conjecture, presumably in view of the fact that, after all these years, a connected vertex-transitive graph without a Hamilton path is yet to be produced. Moreover, only four connected vertex-transitive graphs (having at least three vertices) not having a Hamilton cycle are known to exist: the Petersen graph, the Coxeter graph, and the two graphs obtained from them by replacing each vertex with a triangle. All of these are cubic graphs, suggesting perhaps that no attempt to resolve the above problem can bypass a thorough analysis of cubic vertex-transitive graphs. However, none of these four graphs is a Cayley graph, that is, a vertex-transitive graph with a regular subgroup of automorphisms. This has led to a folklore conjecture that every connected Cayley graph possesses a Hamilton cycle. This problem, together with its Cayley graph variation, has spurred quite a bit of interest in the mathematical community producing, amongst other, conjectures and counterconjectures with regard to its truthfulness. Thomassen [18,82] conjectured that only finitely many connected vertex-transitive graphs without a Hamilton cycle exist, and Babai [15,16] conjectured that infinitely many such graphs exist. More precisely, he conjectured that there exists $\epsilon > 0$ such that there are infinitely many connected vertex-transitive graphs X with longest cycle of length at most $(1-\epsilon)|V(X)|$.

All in all, many articles directly and indirectly related to this subject (see [2–6,8–10,12–14,24,32,37,45,47,53,60,61,63,64,68,69,84–86] for some of the relevant references), have appeared in the literature, affirming the existence of such paths

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and, in some cases, even Hamilton cycles. For example, it is known that connected vertex-transitive graphs of order kp, where $k \leq 4$, (except for the Petersen graph and the Coxeter graph) of order p^j, where $j \leq 4$, and of order $2p^2$, where p is prime, contain a Hamilton cycle. Moreover, it is known that connected vertex-transitive graphs of order pq, where p and q are primes, admitting an imprimitive subgroup of automorphisms contain a Hamilton cycle. Also, a Hamilton path is known to exist in connected vertex-transitive graphs of order $5p$ and $6p$ (see [2,24,27,56,57,62,65,61,63,64,66,83]). As for a general vertex-transitive graph, the best known result is that of Babai, who has shown that a vertex-transitive graph on n vertices has a cycle of length at least $\sqrt{3n}$ [17].

Particular attention has been given to Cayley graphs. Nevertheless, most of the results proved thus far depend on various restrictions made either on the class or order of groups dealt with, or on the generating sets of Cayley graphs (see [10,13,14,43,44,52,78,86]). For example, one may easily see that connected Cayley graphs of abelian groups have a Hamilton cycle. Further, it is known that connected Cayley graphs of hamiltonian groups have a Hamilton cycle (see [13]), and that connected Cayley graphs of metacyclic groups with respect to the standard generating set have a Hamilton cycle (see [5]). Also, following a series of articles [37,53,60] it is now known that every connected Cayley graph of a group with a cyclic commutator subgroup of prime power order, has a Hamilton cycle. This result has later been generalized to connected vertex-transitive graphs whose automorphism groups contain a transitive subgroup with a cyclic commutator subgroup of prime power order, where the Petersen graph is the only counterexample [32]. But perhaps the biggest achievement on the subject is due to Witte (now Morris) who proved that a connected Cayley digraph of any p-group has a Hamilton cycle [86]. On the other hand, even for the class of dihedral groups the question remains open. The best result in this respect is due to Alspach [6] who proved that every connected Cayley graph on a generalized dihedral group of order divisible by 4 has a Hamilton cycle (see also [14]). Three other results of note are a theorem of Witte [84, Theorem 3.1] showing that every group G with minimal generating set of size d contains a generating set of size less than $4d^2$ such that the corresponding Cayley graph has a Hamilton cycle, a theorem of Pak and Radočić [78] showing that every group G has a generating set of size at most $\log_2 |G|$ for which the corresponding Cayley graph has a Hamilton cycle, and a theorem of Krivelevich and Sudakov [55] showing that for every $c > 0$ and large enough n, a Cayley graph formed by choosing a set of $c \log^5 n$ generators randomly from a given group of order n, almost surely has a Hamilton cycle. (For further results not explicitly mentioned or referred here see the survey articles [30,87]).

Various concepts/problems related to Hamilton cycles and paths in vertex-transitive graphs and digraphs, motivated by the original Lovász's question, such as Hamilton connectivity, Hamilton laceability, Hamilton decomposability and edge hamiltonicity, have been studied (see [7,13,25,26,58,88,73,74]). In this article, however, we will only consider the problem of existence of Hamilton paths and cycles in connected vertex-transitive graphs, hereafter refereed to as the *HPC problem*. Also, a graph possessing a Hamilton cycle is said to be *hamiltonian*.

The article is organized as follows. In Section 2 definitions, notation and some auxiliary results are introduced. In Section 3 the main strategies used thus far together with possible future directions in solving the HPC problem are given, in particular, the "lifting Hamilton cycles approach" (see Section 3.1) and the "Hamilton trees on surfaces approach" (see Section 3.2). In addition, the usefulness of general results on the existence of Hamilton cycles in graphs in connection to the HPC problem are also discussed.

2. Notation

Throughout this article graphs are finite, undirected and unless specified otherwise, connected. (In most cases the graphs are simple graphs, however in some instances multiple edges will be allowed.) Given a graph X we let $V(X)$, $E(X)$, $A(X)$ and $\mathrm{Aut}\,X$ be the vertex set, the edge set, the arc set and the automorphism group of X, respectively. A sequence $(u_0, u_1, u_2, \ldots, u_s)$ of distinct vertices in a graph is called an *s-arc* if u_i is adjacent to u_{i+1} for every $i \in \{0, 1, \ldots, s-1\}$. For $S \subseteq V(X)$ we let $X[S]$ denote the induced subgraph of X on S. By an n-cycle we shall always mean a cycle with n vertices. A subgroup $G \leq \mathrm{Aut}\,X$ is said to be *vertex-transitive*, *edge-transitive* and *arc-transitive* provided it acts transitively on the sets of vertices, edges and arcs of X, respectively. A subgroup $G \leq \mathrm{Aut}\,X$ is said to be *s-regular* if it acts regularly on the sets of s-arc of X. The graph X is said to be *vertex-transitive*, *edge-transitive*, and *arc-transitive* if its automorphism group is vertex-transitive, edge-transitive and arc-transitive, respectively. An arc-transitive graph is also called *symmetric*. Given a group G and a subset S of $G \setminus \{1\}$ such that $S = S^{-1}$, the *Cayley graph* $\mathrm{Cay}(G, S)$ has vertex set G and edges of the form $\{g, gs\}$ for all $g \in G$ and $s \in S$. The symbol $\mathbb{Z}_r$ will denote both the cyclic group of order r and the ring of integers modulo r. In the latter case, $\mathbb{Z}_r^*$ will denote the multiplicative group of units of $\mathbb{Z}_r$. By D_{2n} we denote the dihedral group of order $2n$.

Given a transitive group G acting on a set V, we say that a partition $\mathcal{B}$ of V is *G-invariant* if the elements of G permute the parts from $\mathcal{B}$, called *blocks* of G, setwise. If the trivial partitions $\{V\}$ and $\{\{v\} : v \in V\}$ are the only G-invariant partitions of V, then G is said to be *primitive*, and is said to be *imprimitive* otherwise.

For a graph X and a partition $\mathcal{W}$ of $V(X)$, we let $X_{\mathcal{W}}$ be the associated *quotient graph* of X relative to $\mathcal{W}$, that is, the graph with vertex set $\mathcal{W}$ and edge set induced naturally by the edge set $E(X)$. Note that $X_{\mathcal{W}}$ may contain multiple edges. Given integers $k \geq 1$ and $n \geq 2$ we say that an automorphism of a graph is *(k, n)-semiregular* if it has k orbits of length n and no other orbit. In the case when $\mathcal{W}$ corresponds to the set of orbits of a semiregular automorphism $\rho \in \mathrm{Aut}\,X$, the symbol $X_{\mathcal{W}}$ will be replaced by X_ρ. One of the successful strategies in the search for Hamilton cycles in connected vertex-transitive graphs is based on an analysis singling out the structure of the quotient graphs of graphs in question relative to orbits of a semiregular automorphism (see Section 3.1).

We end this section with basic definitions about covering techniques. Let X be a graph, let r be a positive integer, and let $\zeta : A(X) \to S_r$ be a permutation voltage assignment, that is, a function from the set of arcs of X into the symmetric group S_r where reverse arcs carry inverse voltages. We thus have a labeling of the arcs of X by permutations in S_r such that $\zeta_{u,v}\zeta_{v,u} = \mathrm{id}$ for all pairs of adjacent vertices $u, v \in V(X)$, where $\zeta_{u,v}$ denotes the permutation assigned to the arc (u, v). The *covering graph* $\tilde{X} = \mathrm{Cov}(X, \zeta)$ of X *with respect to* ζ has vertex set $V(X) \times \mathbb{Z}_r$, and edges of the form $(u, s)(v, s')$, where $uv \in E(X)$, $s \in \mathbb{Z}_r$ and $s' = s\zeta_{u,v}$. The set of vertices $(u, 0), (u, 1), \ldots, (u, r-1)$ is called the *fibre* of u. The subgroup K of all those automorphisms of $\tilde{X}$ which fix each of the fibres setwise is called the *group of covering transformations*, and the graph $\tilde{X}$ is called a K*-cover* of X. It is a simple observation that, when the covering graph is connected, the group of covering transformations acts semiregularly (that is, every non-trivial element of the group acts without fixed points) on each of the fibres. In particular, if the group of covering transformations is regular on the fibres of $\tilde{X}$, we say that $\tilde{X}$ is a *regular* K*-cover*. In this case, the voltage group, that is, the group generated by $\mathrm{Im}(\zeta)$, is a regular group of degree r abstractly isomorphic to the group of covering transformations K. Given a spanning tree T of the graph X, a voltage assignment ζ is said to be T*-reduced* if the voltages on the tree arcs equal the identity element. In [49] it is shown that every regular cover $\tilde{X}$ of a graph X can be derived from a T-reduced voltage assignment ζ with respect to an arbitrary fixed spanning tree T of X.

3. Current directions in the search for Hamilton cycles and paths

3.1. Lifting Hamilton cycles approach

A frequently used approach to constructing Hamilton cycles in vertex-transitive graphs is based on a quotienting/reduction with respect to an imprimitivity block system of the corresponding automorphism group or, sometimes, with respect to a suitable semiregular automorphism, preferably one of prime order. Provided the quotient graph contains a Hamilton cycle it is sometimes possible to lift this cycle to construct a Hamilton cycle in the original graph, consistently spiraling through the corresponding blocks/orbits (see Example 3.2). This idea was first introduced in [60] to show the existence of Hamilton cycles in Cayley graphs of semidirect products of cyclic groups of prime order with abelian groups of odd order, and was then used (with only slight modifications), following a series of articles, to establish the most general result on this particular subject: existence of Hamilton cycles in connected vertex-transitive graphs with automorphism groups containing a transitive subgroup whose commutator subgroup is cyclic of prime power order, with the Petersen graph being the only counterexample [32,37,53]. We propose to continue this line of research by posing the following problem, a successful solution of which will inevitably shed some new light on the HPC problem for the larger class of solvable groups.

Problem 3.1. Is there a Hamilton cycle/path in a connected vertex-transitive graph whose automorphism group contains a transitive subgroup whose commutator subgroup is an arbitrary cyclic group?

Lifts of Hamilton cycles from quotient graphs which themselves have a Hamilton cycle are always possible, for example, where the quotienting is done relative to a semiregular automorphism of prime order and where in the quotient there are at least two adjacent orbits joined by a double edge. In this case one can always lift the Hamilton cycle from the quotient because the double edge gives us the possibility to conveniently "change direction" so as to get a walk in the quotient that lifts to a full cycle above (see Example 3.2). Sometimes, however, lifts of Hamilton cycles from hamiltonian quotient graphs are possible even if the quotienting is done relative to a semiregular automorphism of non-prime order (see Example 3.3).

This method shows the importance of the semiregularity problem, posed in [67, Problem 2.4], which asks if every vertex-transitive digraph has a semiregular automorphism of prime order. The now commonly accepted, and slightly more general, version of the semiregularity problem involves the whole class of 2-closed transitive groups [22,54]. There has recently been an increased interest in this problem, with a number of articles making small but important steps toward a possible final solution to the problem [23,33,34,41,42,70].

In the example below the well-known *Frucht's notation* [40] for graphs admitting semiregular automorphisms is used. In particular, if X is a connected graph admitting a (k, n)-semiregular automorphism $\rho = (u_0^0 u_0^1 \cdots u_0^{n-1})(u_1^0 u_1^1 \cdots u_1^{n-1}) \cdots (u_{k-1}^0 u_{k-1}^1 \cdots u_{k-1}^{n-1})$, with the set of orbits $\mathcal{W} = \{W_i \mid i \in \mathbb{Z}_k\}$, where $W_i = \{u_i^s \mid s \in \mathbb{Z}_n\}$, then using Frucht's notation [40] the graph X may be represented in the following way. Each orbit of ρ is represented by a circle. Inside a circle corresponding to the orbit W_i the symbol n/T, where $T = T^{-1} \subseteq \mathbb{Z}_n \setminus \{0\}$, indicates that for each $s \in \mathbb{Z}_n$, the vertex u_i^s is adjacent to all the vertices u_i^{s+t} where $t \in T$. When $|T| \leq 2$ we use a simplified notation n/t, $n/(n/2)$ and n, respectively, when $T = \{t, -t\}$, $T = \{n/2\}$ and $T = \emptyset$. Finally, an arrow pointing from the circle representing the orbit W_i to the circle representing the orbit W_j, $j \neq i$, labeled by $y \in \mathbb{Z}_n$ means that for each $s \in \mathbb{Z}_n$, the vertex $u_i^s \in W_i$ is adjacent to the vertex u_j^{s+y}. When the label is 0, the arrow on the line may be omitted. When there are several arrows pointing from the circle representing the orbit W_i to the circle representing the orbit W_j, $j \neq i$, these arrows may be represented by a single arrow with multiple labels. The description of a graph using Frucht's notation corresponds to the fact that such a graph is a cyclic cover of the corresponding quotient with respect to a semiregular automorphism. The various above mentioned labels in circles and on arrows correspond to respective voltages.

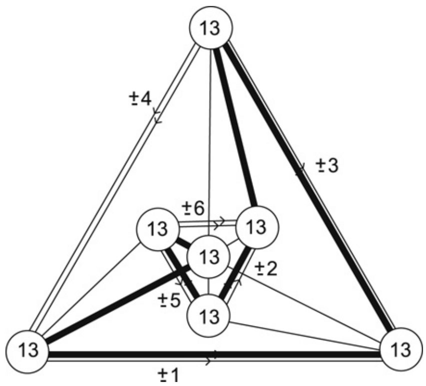

Fig. 1. A vertex-transitive graph arising from the action of PSL(2, 13) on cosets of D_{12} given in Frucht's notation with respect to the (9, 13)-semiregular automorphism ρ where undirected lines carry label 0. Edges in bold show a Hamilton cycle.

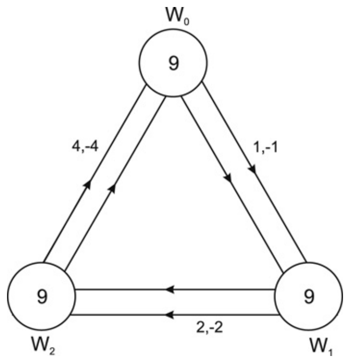

Fig. 2. The Holt graph given in Frucht's notation with respect to the (3, 9)-semiregular automorphism ρ.

Example 3.2. The orbital graph X arising from the action of PSL(2, 13) on cosets of D_{12} with respect to a self-paired suborbit of length 6 (see [71, page 198] for details) contains a (7, 13)-semiregular automorphism ρ, and it can be nicely represented in Frucht's notation as shown in Fig. 1. Since the quotient graph X_ρ has a Hamilton cycle containing a double edge and since 13 is a prime number, this cycle can be lifted to a Hamilton cycle in the original graph X (see Fig. 1).

Example 3.3. The Holt graph X, the smallest half-arc-transitive graph (see [11,35,50]), has vertex set $V(X) = \{u_i^j \mid i \in \mathbb{Z}_3, j \in \mathbb{Z}_9\}$ and edge set $E(X) = \{u_i^j u_{i+1}^{j\pm 2^i} \mid i \in \mathbb{Z}_3, j \in \mathbb{Z}_9\}$. Clearly, the permutation ρ defined by the rule $u_i^j\rho = u_i^{j+1}$, where $i \in \mathbb{Z}_3$ and $j \in \mathbb{Z}_9$, is a (3, 9)-semiregular automorphism of X and its orbits are $W_0 = \{u_0^j \mid j \in \mathbb{Z}_9\}$, $W_1 = \{u_1^j \mid j \in \mathbb{Z}_9\}$, and $W_2 = \{u_2^j \mid j \in \mathbb{Z}_9\}$ (see Fig. 2). The quotient graph X_ρ with respect to the (3, 9)-semiregular automorphism ρ has three vertices, corresponding to the three orbits W_0, W_1 and W_2 of ρ, and any two vertices in X_ρ are joined by a double edge. Since $1 + 2 + 4 = 7$ generates $\mathbb{Z}_9$ the Hamilton cycle $W_0W_1W_2$ of X_ρ lifts to a Hamilton cycle in the Holt graph X: $u_0^0u_1^1u_2^3u_0^7u_1^8u_2^0 \ldots u_1^3u_2^5u_0^0$.

The existence of Hamilton cycles in connected vertex-transitive graphs can sometimes be shown using results from classical graph theory. Let us stress the result, due to Jackson [51], giving a sufficient condition for the existence of Hamilton cycles in regular graphs. In particular, Jackson's result says that every 2-connected regular graph of order n and valency at least $n/3$ is hamiltonian. Since connected vertex-transitive graphs are 2-connected and regular this result implies that in order to solve the HPC problem it is sufficient to consider connected vertex-transitive graphs of 'small' valency. Also, some of the research done on the existence of Hamilton cycles in connected vertex-transitive graphs shows the usefulness of the lifting Hamilton cycle approach when combined with classical results on hamiltonian graphs such as the result of Nash-Williams that k-regular graphs on $2k + 1$ vertices are hamiltonian [75] as well as the well-known Dirac's theorem [31] and Ore's theorem [77] (see also Example 3.5). In particular, the following remarkable result of Chvátal on Hamilton's ideals [28] is useful in this respect. Hamilton's ideals are the sets S_i in the proposition below.

Proposition 3.4 *([28]). Let X be a graph and let $S_i = \{x \in V(X) \mid deg(x) \le i\}$. Then X has a Hamilton cycle if for each $i < n/2$ either $|S_i| \le i - 1$ or $|S_{n-i-1}| \le n - i - 1$.*

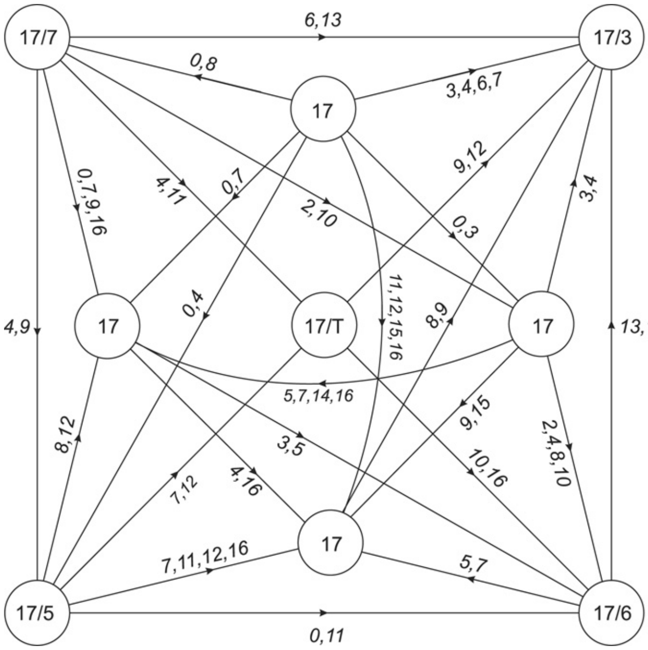

Fig. 3. A vertex-transitive graph arising from the action of PSL(2, 17) on cosets of D_{16} given in Frucht's notation with respect to a (9, 17)-semiregular automorphism, where $T = \{\pm 3, \pm 5, \pm 6, \pm 7\}$.

Example 3.5. The orbital graph X of valency 16 arising from the action of PSL(2, 17) on cosets of D_{16} with respect to a self-paired suborbit of length 16 is of order 153 and it contains a (9, 17)-semiregular automorphism ρ. It can be given in Frucht's notation with respect to ρ as shown in Fig. 3. The simplified quotient graph with respect to ρ (obtained by forgetting multiple edges) has one vertex of valency 4 and eight vertices of valency 6. Thus Ore's theorem implies that X_ρ is hamiltonian. Also, since all the edges in X_ρ are multiple edges, the lifting method enable us to construct a Hamilton cycle in X.

For further Chvátal type results on the existence of Hamilton cycles not explicitly mentioned here we refer the reader to [19,46,81]. The next example illustrates how Proposition 3.4 can be applied to show the existence of Hamilton cycles in a vertex-transitive graph, an orbital graph arising from the action of PSL(2, p) on the cosets of D_{p-1} with respect to a self-paired suborbit of length $p-1$. Finding Hamilton cycles in this graph is part of a wider project aimed at proving the existence of Hamilton cycles in connected vertex-transitive graphs of order a product of two odd primes [36].

Example 3.6. Let $G = \mathrm{PSL}(2, p)$, and let $H \le G$ be its subgroup isomorphic to D_{p-1}, where $p \ge 17$ is a prime such that $q = (p+1)/2$ is also a prime. Let $F = GF(p)$ and $F^* = F \setminus \{0\}$. Let S^* denote the set of all non-zero squares in F and let $N^* = F^* \setminus S^*$. For simplicity we refer to the elements of G as matrices. Then H consists of all the matrices of the form

$$\begin{bmatrix} x & 0 \\ 0 & x^{-1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & -x \\ x^{-1} & 0 \end{bmatrix} \quad (x \in F^*),$$

and the vertex-transitive graph arising from the action of G on the set $\mathcal{H}$ of right cosets of H can be described in the following way. (For further details see [72].) Firstly, note that in order for $q = (p+1)/2$ to be a prime we must have $p \equiv 1 \pmod 4$. Secondly, for a typical element $g = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ of G we let $\xi(g) = ad$ and $\eta(g) = a^{-1}b$. Further we let $\chi(g) = (\xi(g), \eta(g))$. Let $\sim$ be the equivalence relation on $F \times F^*$ defined by $(\xi, \eta) \sim (1-\xi, \frac{\xi\eta}{\xi-1})$ for $\xi \ne 0, 1$. There is then a natural identification of the sets $\mathcal{H}$ and $(F \times F^*)/_\sim \cup \{\infty\}$ where ∞ corresponds to H and (ξ, η) corresponds to the coset Hg satisfying $\chi(g) = (\xi, \eta)$.

For each $\xi \in F^*$ define the following subsets of $\mathcal{H}$: $\mathcal{S}_\xi^+ = \{(\xi, \eta) : \eta \in S^*\}$ and $\mathcal{S}_\xi^- = \{(\xi, \eta) : \eta \in N^*\}$. From [72], where all the suborbits of the action of G on $\mathcal{H}$ are determined, we can extract that $\mathcal{S}_\xi = \mathcal{S}_\xi^+ \cup \mathcal{S}_\xi^-$, where $\xi(1-\xi) \in N^*$, is a self-paired suborbit of length $p-1$. Let $X = X(G, H, \mathcal{S}_\xi)$ be the corresponding orbital graph, that is, the graph of order $pq = p(p+1)/2$ with vertex set $V(X) = \{Hg \mid g \in G\}$ and edge set $E(X) = \{\{Hg, Hsg\} \mid g \in G, s \in \mathcal{S}_\xi\}$. Then the Sylow p-subgroup of G generated by the matrix $\rho = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ acts on X semiregularly with q orbits of size p. Let V_∞ and V_x be the orbit of ρ containing, respectively, the coset H and the coset $H \begin{bmatrix} 1 & x \\ x^{-1} & 1 \end{bmatrix}$, where $x \in F^*$. A short computation shows that $V_x = V_{-x}$. Now consider the quotient graph X_ρ. By computation, we get that for every $x \in F^*$ the orbit V_x induces a graph of valency 2, and that there exists a double edge between V_∞ and any other of the remaining $(p-1)/2$ vertices in X_ρ. If the

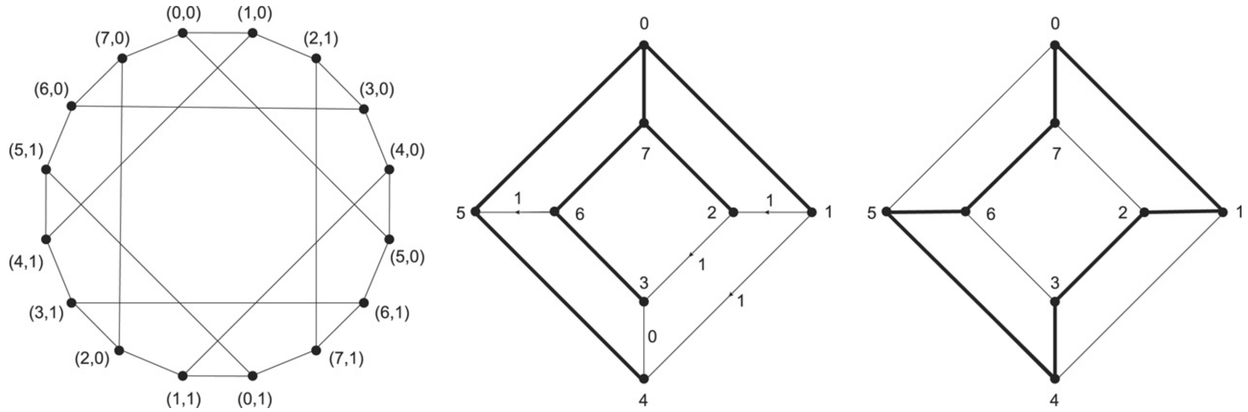

Fig. 4. The Moebius–Kantor graph GP(8, 3), T-reduced voltage assignment in the cube giving rise to the Moebius–Kantor graph and a Hamilton cycle in the cube.

subgraph $X_\rho - \{V_\infty\}$ of X_ρ induced by $V(X_\rho) \setminus \{V_\infty\}$ is regular then it turns out that in the simplified graph of $X_\rho - \{V_\infty\}$ (obtained by forgetting multiple edges) the vertices $V_x, x \in F^*$, are all of valency greater than or equal to $(p-5)/4$ which is greater than $(p+1)/6$, since $p \geq 17$. Hence, Jackson's result implies that the simplified graph of $X_\rho - \{V_\infty\}$ and therefore $X_\rho - \{V_\infty\}$ itself are hamiltonian. Moreover, the fact that V_∞ is connected to every other vertex in X_ρ, implies that also X_ρ is hamiltonian. On the other hand, if $X_\rho - \{V_\infty\}$ is not regular then further computations show that in the simplified graph of X_ρ there are $(p-1)/4$ vertices of valency $(p-1)/4$, next $(p-1)/4$ vertices of valency $(p+3)/4$ and one vertex of valency $(p-1)/2$. Since X_ρ is of order $q = (p+1)/2$, Proposition 3.4 implies that X_ρ is hamiltonian. Now using the fact that V_∞ in X_ρ is connected to every other vertex with a double edge and applying the lifting approach we can construct a Hamilton cycle in X.

All of the above comments also suggest that the essential question that should be addressed in the context of lifts of Hamilton cycles concerns the case when any two adjacent orbits in the quotient are joined by a single edge. In other words, the question is under what circumstances is a covering graph of a hamiltonian graph also hamiltonian. The importance of this general question lies, for example, in the fact that these sorts of problems are encountered in many special cases when one is searching for Hamilton cycles. As an illustration, Witte's theorem about Hamilton cycles in Cayley (di)graphs of p-groups [86] could be successfully generalized to arbitrary vertex-transitive graphs of prime power order provided one could prove that for a prime p, a connected regular $\mathbb{Z}_p$-cover of a hamiltonian vertex-transitive graph of order a power of p, is hamiltonian. Then an inductive argument along these lines could be applied. Let X be a connected vertex-transitive graph of order p^k and assume that every connected vertex-transitive graph of order p^{k-1} is hamiltonian. Let P be a Sylow p-subgroup of Aut X. Then P acts transitively on $V(X)$. Let $\rho \in Z(P)$ be a central element in P of prime order. We then consider the quotient X_ρ of X relative to the set of orbits of ρ. This, being a connected vertex-transitive graph of prime power order smaller than p^k, may be assumed to be hamiltonian. We now consider two possibilities depending on whether the orbits of ρ are independent sets or not. If not we can find a Hamilton cycle in X as a cycle in a Cartesian product of two cycles. If however the orbits are independent sets then we may assume that X is a cover of X_ρ and we are left with the following problem.

Problem 3.7. Is a connected regular $\mathbb{Z}_p$-cover, where p is a prime, of a hamiltonian vertex-transitive graph of order p^k hamiltonian?

Example 3.8. The Moebius–Kantor graph, also known as the generalized Petersen graph GP(8, 3), shown in the left-hand side picture in Fig. 4, is the unique cubic symmetric graph of order 16. It is a regular $\mathbb{Z}_2$-cover of the cube, and it can be reconstructed via the T-reduced voltage assignment in the cube given in the middle picture in Fig. 4 (see [39]). The Hamilton cycle in the cube, shown in the right-hand side picture in Fig. 4, has voltage 1, and therefore gives rise to a Hamilton cycle in the Moebius–Kantor graph.

Motivated by Example 3.8 we propose the following question addressing $\mathbb{Z}_2$-covers of arbitrary hamiltonian vertex-transitive graphs.

Problem 3.9. Is a connected regular $\mathbb{Z}_2$-cover of a hamiltonian vertex-transitive graph also hamiltonian?

3.2. Hamilton trees on surfaces approach

This approach for finding Hamilton cycles/paths, although in theory applicable for vertex-transitive graphs of any valency, has proved useful in particular for cubic Cayley graphs. To introduce this approach we need to define cycle-separating sets

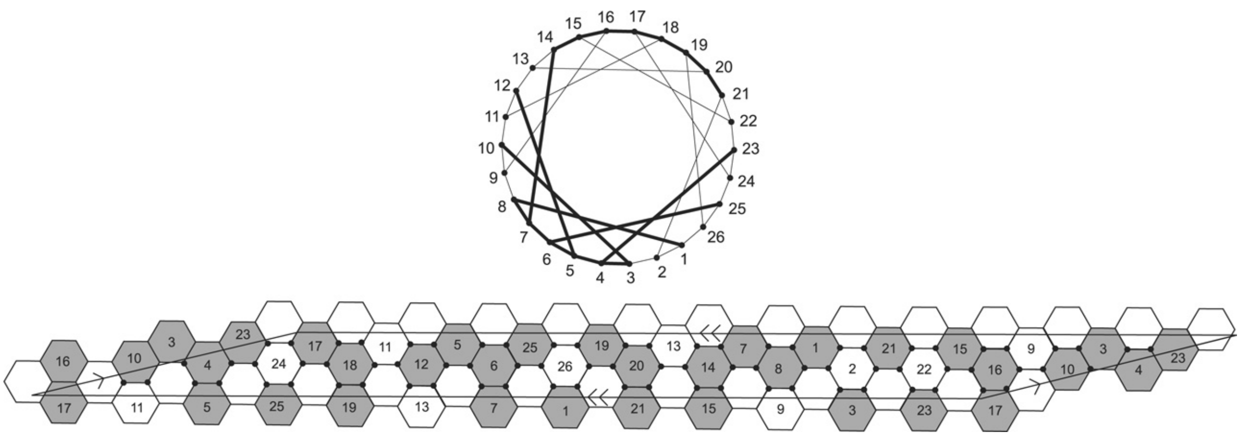

Fig. 5. A Hamilton tree of faces in a toroidal Cayley map of a Cayley graph of $\mathbb{Z}_{13} \rtimes \mathbb{Z}_6$ with respect to a (2, 6, 3)-presentation giving rise to a Hamilton cycle, and the associated hexagon graph.

and cyclic edge connectivity. Given a connected graph X, a subset $F \subseteq E(X)$ of edges of X is said to be *cycle-separating* if $X - F$ is disconnected and at least two of its components contain cycles. We say that X is *cyclically k-edge-connected* if no set of fewer than k edges is cycle-separating in X. Following [79] we say that, given a graph (or more generally a loopless multigraph) Y, a subset S of $V(Y)$ is *cyclically stable* if the induced subgraph $X[S]$ is acyclic, that is, a forest. The following result of Payan and Sakarovitch [79, Théorème 5] on cyclically stable subsets in cyclically 4-edge-connected cubic graphs was essential in proving the existence of Hamilton paths/cycles in a particular family of cubic Cayley graphs [44].

Proposition 3.10 (*[79]*). *Let X be a cyclically 4-edge-connected cubic graph of order n, and let S be a maximum cyclically stable subset of* $V(X)$. *Then* $|S| = \lfloor (3n-2)/4 \rfloor$ *and more precisely, the following hold.*

(i) *If* $n \equiv 2 \pmod 4$ *then* $|S| = (3n-2)/4$, *and* $X[S]$ *is a tree and* $V(X) \setminus S$ *is an independent set of vertices.*
(ii) *If* $n \equiv 0 \pmod 4$ *then* $|S| = (3n-4)/4$, *and either* $X[S]$ *is a tree and* $V(X) \setminus S$ *induces a graph with a single edge, or* $X[S]$ *has two components and* $V(X) \setminus S$ *is an independent set of vertices.*

As mentioned in Section 1, the four non-hamiltonian vertex-transitive graphs are all cubic, thus suggesting that cubic vertex-transitive graphs are the natural first candidates for which the HPC problem needs to be tested. A promising first step in this direction was made in [44] where, with an innovative approach, a Hamilton path was shown to exist in cubic Cayley graphs arising from quotients of the modular group $PSL(2, \mathbb{Z})$, that is, from finite groups $G = \langle a, x \mid a^2 = 1, x^s = 1, (ax)^3 = 1, \ldots \rangle$ having a $(2, s, 3)$-presentation. The strategy is based on an embedding of a Cayley graph $X = \mathrm{Cay}(G, S)$, $S = \{a, x, x^{-1}\}$, onto the closed orientable surface of genus $g = 1 + (s-6)|G|/12s$ whose faces are $|G|/s$ disjoint s-gons and $|G|/3$ hexagons. This map is given by using the same rotation of the x, a, x^{-1} edges at every vertex and results in one s-gon and two hexagons adjacent at each vertex. (An embedding of a Cayley graph onto an oriented surface having the same cyclic rotation of generators around each vertex is called a *Cayley map*, see [80].) In this map one then looks for a long tree of faces—a tree of faces whose boundary is either a Hamilton cycle in X or a cycle missing two adjacent vertices of X. Although similar methods have been used before to find Hamilton cycles in certain Cayley graphs in [27,29,45], the new and essential ingredient in the proof of the above result ties the search for a long tree of faces, sought after in the corresponding Cayley map, with a classical result of Payan and Sakarovitch [79] regarding decompositions of cyclically 4-edge-connected cubic graphs into induced trees whose complements have at most one edge (see Proposition 3.10). Namely, we associate with the Cayley graph X the so called *Hexagon graph* $\mathrm{Hex}(X)$, whose vertex set consists of the hexagons arising from the relation $(ax)^3 = 1$, with two hexagons being adjacent if they share an edge. Of course, G acts regularly on the arcs of $\mathrm{Hex}(X)$ with a cyclic vertex stabilizer. A long tree of faces in the Cayley map of X then arises from sufficiently large induced trees in $\mathrm{Hex}(X)$ which are proved to exist using a delicate argument, interweaving the above result of Payan and Sakarovitch, a careful analysis of cubic arc-transitive graphs with small girth, and a result of Nedela and Škoviera [76] which says that cyclic edge connectivity of a cubic connected vertex-transitive graph equals its girth (see Example 3.11).

Example 3.11. In the bottom picture in Fig. 5 we show a tree of hexagons whose boundary is a Hamilton cycle in the toroidal Cayley map of the Cayley graph of the group $G = \mathbb{Z}_{13} \rtimes \mathbb{Z}_6 = \langle y, z \mid y^6 = z^{13} = 1, z^y = z^4 \rangle$ with respect to a $(2, 6, 3)$-presentation $\langle a, x \mid a^2 = x^6 = (ax)^3 = 1, \ldots \rangle$, where $a = y^3$ and $x = y^3 z^{-1} y^2 z$. The upper picture shows this same tree in the associated hexagon graph, the only cubic symmetric graph of order 26 known as the graph F026A (see [20]). The correspondence between hexagons in the Cayley map and the vertices in the hexagon graph is indicated by numbers from 1 to 26. Observe that when the group G is considered as a subgroup of the symmetric group S_{13}, its $(2, 6, 3)$-presentation is defined by $a := (1\,7)(2\,6)(3\,5)(8\,13)(9\,12)(10\,11)$ and $x := (1\,11\,7\,6\,9\,13)(2\,8\,3\,5\,12\,4)$.

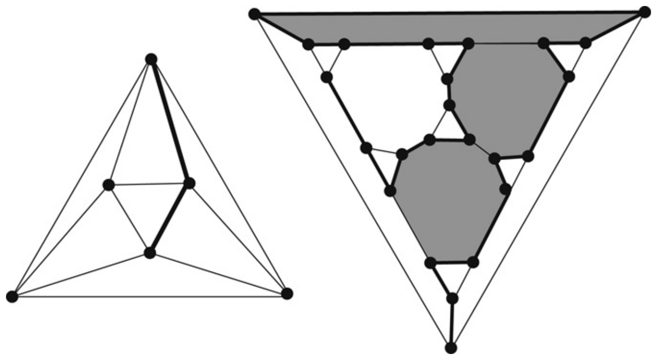

Fig. 6. A tree of faces in the spherical Cayley map of a Cayley graph of S_4 with respect to a (2, 3, 4)-presentation giving rise to a cycle missing two pairs of adjacent vertices, and the associated octagon graph.

This method always gives us a Hamilton cycle when the associated hexagon graph is of order congruent to 2 modulo 4 since in this case the hexagon graph possesses an induced tree whose complement is an independent set of vertices (see Proposition 3.10). On the other hand, when the hexagon graph is of order congruent to 0 modulo 4, no such induced tree exists, and consequently this approach only ensures the existence of Hamilton paths.

Current work on the problem suggests that this particular approach may indeed bear fruit. First, despite a number of technical obstacles encountered along the way, the method has already been successfully refined to produce a full Hamilton cycle in every Cayley graph of a group with a $(2, s, 3)$-presentation for s divisible by 4 (see [43]), with the last details also being worked out in the remaining cases s odd and s congruent to 2 modulo 4. Second, there may very well be ways for possible generalizations to arbitrary cubic Cayley graphs arising from groups with generating sets consisting of an involution and a non-involution of order s whose product is of order $t \geq 4$, in short, from groups with a $(2, s, t)$-presentation, $t \geq 4$. In this case, an arc-transitive graph of valency t with a cyclic vertex stabilizer whose vertex set consists of all the $2t$-gons in the corresponding Cayley map, an analogue of the Hexagon graph, is associated with the Cayley graph in question, suggesting that a search for Hamilton paths/cycles will depend on a further study of decompositions of regular graphs, and in particular arc-transitive graphs of valency $t \geq 4$ with cyclic vertex stabilizers, into induced forests whose complements are independent sets or have a small number of edges (see Example 3.14). This opens up a new area of research, interesting in its own right, not just as a means for constructing Hamilton paths/cycles or, at least, relatively long paths/cycles in cubic Cayley graphs. In summary, we propose the following problems.

Problem 3.12. Let $G = \langle a, x \mid a^2 = 1, x^s = 1, (ax)^t = 1, \ldots\rangle$ be a group having a $(2, s, t)$-presentation. Is the Cayley graph $\mathrm{Cay}(G, S)$, where $S = \{a, x, x^{-1}\}$, hamiltonian?

Given a graph X a subgroup G of the automorphism group $\mathrm{Aut}\,X$ is said to be 1-regular if it acts regularly (that is, transitively with a trivial stabilizer) on the arc set of X.

Problem 3.13. Let X be an arc-transitive graph of valency greater than or equal to 4 admitting a 1-regular subgroup of automorphisms with cyclic vertex stabilizer. Decompose the vertex set $V(X)$ into an induced tree whose complement induces a graph with as few edges as possible. (By Proposition 3.10, for cubic graphs such a complement induces a graph with at most one edge.)

Example 3.14. In the right-hand side picture in Fig. 6 we show a tree of octagons whose boundary is a cycle missing only two pairs of adjacent vertices in the spherical Cayley map of a Cayley graph X of the group S_4 with a (2, 3, 4)-presentation $\langle a, x \mid a^2 = x^3 = (ax)^4 = 1, \ldots\rangle$, where $a = (1\ 2)$ and $x = (1\ 3\ 4)$. The left-hand side picture shows this same tree in the corresponding "octagon" graph, tetravalent arc-transitive graph. Starting at one of the four vertices that are not contained in the tree of octagons in X, then moving to its neighbor that is also not contained in the boundary of the tree, and continuing to the boundary of the tree, following this boundary until all vertices on the boundary have been visited, and finally going to the other two adjacent vertices not contained on the boundary of the tree, yields a Hamilton path in X (see Fig. 6).

The following example shows that this and similar approaches may be successful in the context of more general families of cubic Cayley graphs, and perhaps cubic vertex-transitive graphs in general.

Example 3.15. In the right-hand side picture in Fig. 7 we show a tree of hexagons whose boundary is a cycle missing only two adjacent vertices in the toroidal Cayley map of a cubic Cayley graph X of the dihedral group D_{48} with respect to the generating set $S = \{x, xy^3, xy^{11}\}$, where $D_{48} = \langle x, y \mid x^2 = y^{24} = (xy)^2 = 1\rangle$. Clearly this cycle gives rise to a Hamilton path in X. Namely, starting at one of the two missing vertices, then going through the other missing vertex, and finally following the boundary of the tree of hexagons yield a Hamilton path in X. In the left-hand side and the middle picture this same tree is shown in the corresponding "hexagon" graph, the Moebius–Kantor graph GP(8, 3).

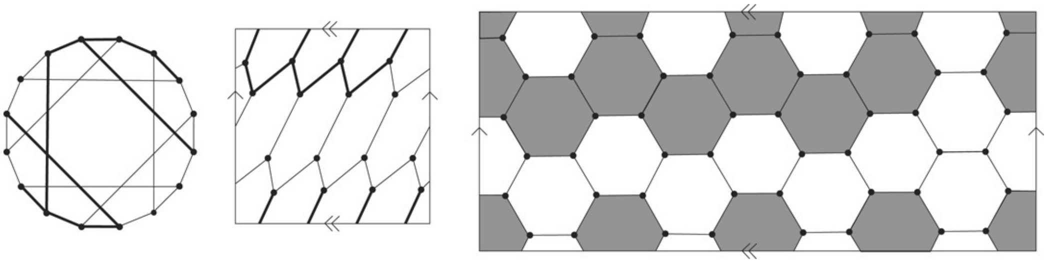

Fig. 7. A tree of faces in the toroidal Cayley map of a Cayley graph of D_{48} with respect to a generating set consisting of three involutions missing two adjacent vertices, and the corresponding induced tree in the Moebius–Kantor graph GP(8, 3) shown in the plane and on the torus.

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