

Nodal solutions in a general Moore–Nehari problem: a dynamic and bifurcation approach

Eduardo Muñoz-Hernández
Complutense University of Madrid

In this talk, we will analyze the existence and multiplicity of nodal solutions for a class of Sturm–Liouville boundary value problems associated to second order nonlinear ODE’s of the type

$$-(\phi(u'))' = \lambda u + a(t)g(u) \quad \text{in } (0, L), \quad (1)$$

where $\lambda \in \mathbb{R}$, ϕ an increasing homeomorphism, with some minimal regularity constraints, g a continuous function such that $g(u)u > 0$, and $a \gneq 0$ is a piecewise constant function. More precisely, we assume that $[0, L]$ splits out into finitely many intervals where, alternately, either a is a positive constant or $a \equiv 0$. The motivation for such a choice for $a(t)$ goes back to the pioneering work of Moore and Nehari (1959). Then, it will be performed a phase plane analysis in the Sturm–Liouville boundary value problem associated to (1). Namely, by composing appropriately the Poincaré maps associated to each of the subintervals of $[0, L]$, and according to the sign of λ , we will deliver a series of multiplicity results together with a rather precise description of the nodal structure of the solutions.

As an application of the abstract results we will focus attention into the simpler, but paradigmatic, Dirichlet problem

$$\begin{cases} -u'' = \lambda u + a(t)|u|^{p-1}u & \text{in } (0, L), \\ u(0) = u(L) = 0, \end{cases} \quad (2)$$

where the global bifurcation diagrams of nodal solutions in terms of the parameter λ will be also ascertained for both the superlinear case $p > 1$ and the sublinear case $0 < p < 1$ when $a \gneq 0$ is a piecewise continuous function.

This is a joint work with Prof. Julián López-Gómez (Complutense University of Madrid, Spain) and Prof. Fabio Zanolin (Udine University, Italy).